STR 665
RISK MANAGEMENT

LECTURE 3: THE ANALYTICAL HIERARCHY PROCESS (AHP)

ANNOUNCEMENT

- Project on website

INTRODUCTION

- In its general form the AHP is a nonlinear framework for carrying out both deductive and inductive thinking without use of the syllogism by taking several factors into consideration simultaneously and allowing for dependence and for feedback, and making numerical trade-offs to arrive at a synthesis or conclusion.

Steps of the AHP

(1) Develop a model for the decision

(2) Derive priorities (weights) for the criteria

(3) Derive local priorities (preferences) for the alternatives

(4) Derive Overall Priorities (Model Synthesis)

(5) Perform Sensitivity analysis

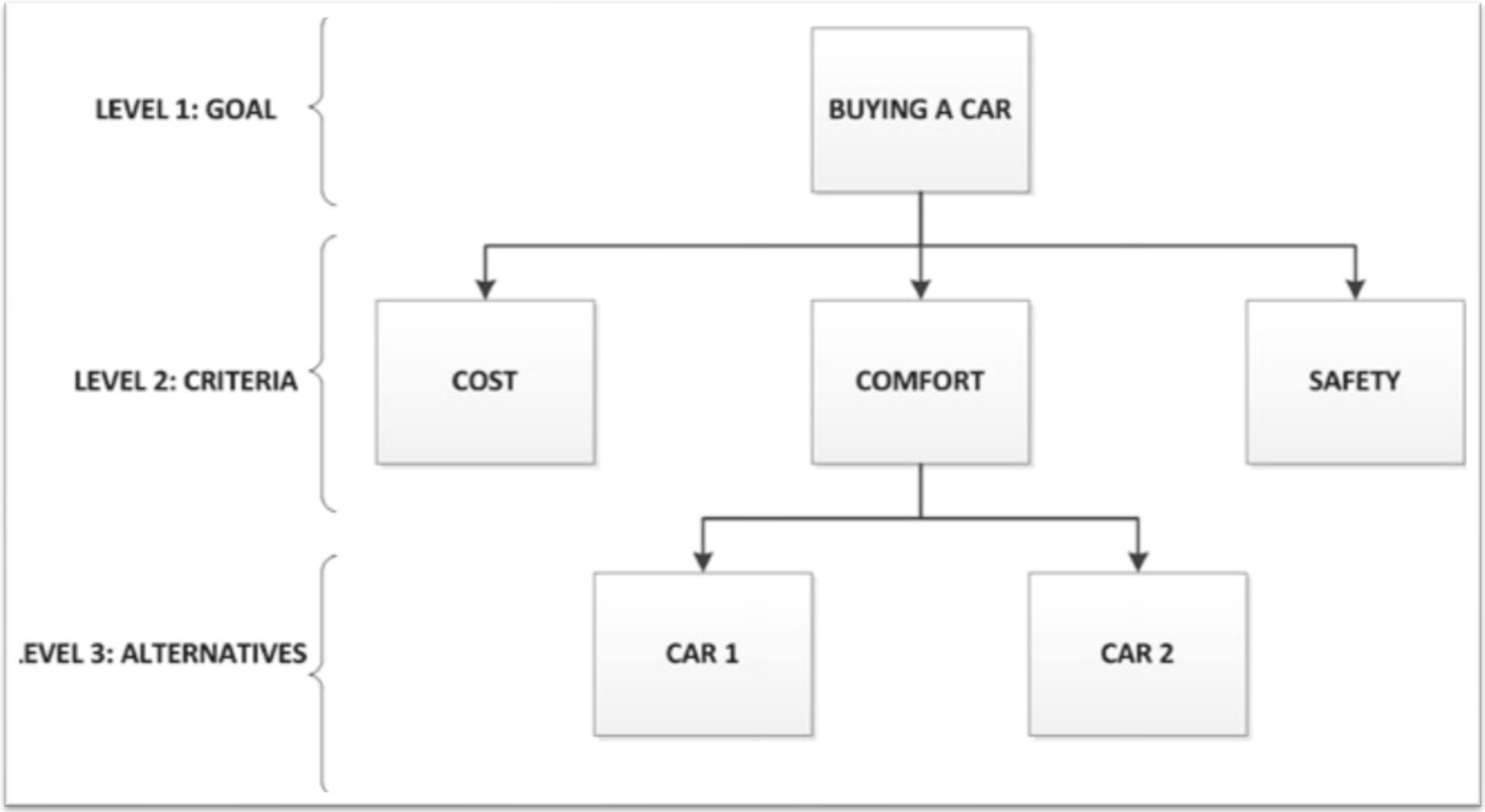
(6) Making a Final Decision

1. Developing a Model

Break down the decision into a hierarchy of

- 1. goals,
- 2. criteria, and
- 3. alternatives.

1. Developing a Model



1. Developing a Model

By structuring the problem in this way it is possible to better understand the decision to be achieved, the criteria to be used and the alternatives to be evaluated.

2. Deriving Priorities (Weights) for the Criteria

derive by pairwise comparisons the relative priority of each criterion with respect to each of the others using a numerical scale for comparison developed by Saaty (2012)

Intensity of Importance	*Definition*	*Explanation*
1	Equal Importance	Two activities contribute equally to the objective
2	Weak or slight	
3	Moderate importance	Experience and judgement slightly favour one activity over another
4	Moderate plus	
5	Strong importance	Experience and judgement strongly favour one activity over another
6	Strong plus	
7	Very strong or demonstrated importance	An activity is favoured very strongly over another; its dominance demonstrated in practice
8	Very, very strong	
9	Extreme importance	The evidence favouring one activity over another is of the highest possible order of affirmation
Reciprocals of above	If activity *i* has one of the above non-zero numbers assigned to it when compared with activity *j*, then *j* has the reciprocal value when compared with *i*	A reasonable assumption
1.1–1.9	If the activities are very	May be difficult to assign the best value but

2. Deriving Priorities (Weights) for the Criteria

Not all criteria are equally important in a given time

- → derive by pairwise comparisons the relative priority of each criterion with respect to each of the others using a numerical scale for comparison developed by Saaty

2. Deriving Priorities (Weights) for the Criteria

Buying a car	Cost	Comfort	Safety
Cost			
Comfort			
Safety			

2. Deriving Priorities (Weights) for the Criteria

- Pairwise comparison matrix with judgments

Buying a car	Cost	Comfort	Safety
Cost	1	7	3
Comfort	1/7	1	1/3
Safety	1/3	3	1

(3) Derive local priorities (preferences) for the alternatives

CALCULATE THE OVERALL PRIORITIES OR WEIGHTS

- Using the approximate method:
  1. Normalize the comparison matrix (add the values in each column)
  2. divide each cell by the total of the column
  3. obtain the overall or final priorities by simply calculating the average value of each row

Buying a car	Cost	Comfort	Safety
Cost	1	7	3
Comfort	1/7	1	1/3
Safety	1/3	3	1

1. Add values in each column

Buying a car	Cost	Comfort	Safety
Cost	1.000	7.000	3.000
Comfort	0.143	1.000	0.333
Safety	0.333	3.000	1.000
Sum	1.476	11.000	4.333

2. divide each cell by the total of the column

Buying a car	Cost	Comfort	Safety
Cost	0.677	0.636	0.692
Comfort	0.097	0.091	0.077
Safety	0.226	0.273	0.231

3. calculate the average value of each row

Buying a car	Cost	Comfort	Safety	Priority
Cost	0.677	0.636	0.692	**0.669**
Comfort	0.097	0.091	0.077	**0.088**
Safety	0.226	0.273	0.231	**0.243**

3. Measuring Consistency

- In a comparison matrix criteria, if we provide a value of 2 to the first criterion over the second and a assign a value of 3 to the second criterion with respect to the third one,

What is the value of preference of the first criterion with respect to the third one?

- Some inconsistency is expected and allowed in AHP analysis.

CONSISTENCY INDEX

- Since the numeric values are derived from the subjective preferences of individuals, it is impossible to avoid some inconsistencies in the final matrix of judgments.
- The question is how much inconsistency is acceptable ?
- AHP calculates a consistency ratio (CR) comparing the consistency index (CI) of the matrix in question (the one with our judgments) versus the consistency index of a random-like matrix (RI).

$$CR = CI/RI$$

- RI is the average CI of 500 randomly filled in matrices.

CONSISTENCY INDEX

n	3	4	5	6
RI	0.58	0.9	1.12	1.24

- A consistency ratio (CR) of 0.10 or less is acceptable to continue the AHP analysis.
- If the consistency ratio is greater than 0.10, it is necessary to revise the judgments to locate the cause of the inconsistency and correct it.

CALCULATING CI

- 1. For matrix showing the judgment comparisons and derived priorities
- 2. Use the priorities as **factors (weights)** for each column

Buying a car	Cost	Comfort	Safety
Criteria Weights ->	*0.669*	*0.088*	*0.243*
Cost	1.000	7.000	3.000
Comfort	0.143	1.000	0.333
Safety	0.333	3.000	1.000

CALCULATING CI

- 1. For matrix showing the *judgment comparisons* and *derived priorities*
- 2. Use the priorities as **factors (weights)** for each column
- 3. Multiply each value in the first column of the comparison matrix by the first criterion priority (weighted columns)

Buying a car	Cost	Comfort	Safety
Cost	0.669	0.617	0.729
Comfort	0.096	0.088	0.081
Safety	0.223	0.265	0.243

CALCULATING CI

- 1. For matrix showing the *judgment comparisons* and *derived priorities*
- 2. Use the priorities as factors (weights) for each column
- 3. Multiply each value in the first column of the comparison matrix by the first criterion priority (weighted columns)
- 4. Add the values in each row to obtain a set of values called **weighted sum**

Buying a car	Cost	Comfort	Safety	Weighted sum
Cost	0.669	0.617	0.729	2.015
Comfort	0.096	0.088	0.081	0.265
Safety	0.223	0.265	0.243	0.731

CALCULATING CI

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- 4. Add the values in each row to obtain a set of values called weighted sum
- 5. Divide the elements of the *weighted sum* vector by the *corresponding priority* of each criterion

Weighted sum	Priority	
2.015/	0.669 =	3.014
0.265/	0.088 =	3.002
0.731/	0.243 =	3.005

CALCULATING CI

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- 4. Add the values in each row to obtain a set of values called weighted sum
- 5. Divide the elements of the *weighted sum* vector by the *corresponding priority* of each criterion
- 6. Calculate the average of the values from the previous step; this value is called ηmax.

Weighted sum	Priority	
2.015/	0.669 =	3.014
0.265/	0.088 =	3.002
0.731/	0.243 =	3.005
	Total	9.021
	Divide Total by 3 to obtain $Lambda_{max}$ =	3.007

CALCULATING CI

- 1. For matrix showing the *judgment comparisons* and *derived priorities*
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- 3. Multiply each value in the first column of the comparison matrix by the first criterion priority (weighted columns)
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- 7. calculate the consistency index (CI)

$$\text{C.I.} = (\lambda_{\max} - n)/(n-1)$$

CALCULATING CI

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$$\text{CI} = (\lambda_{\max} - n)/(n-1) = (3.007-3)/(3-1) = \mathbf{0.004}$$

CALCULATING CI

- 1. For matrix showing the *judgment comparisons* and *derived priorities*
- 2. Use the priorities as factors (weights) for each column
- 3. Multiply each value in the first column of the comparison matrix by the first criterion priority (weighted columns)
- 4. Add the values in each row to obtain a set of values called weighted sum
- 5. Divide the elements of the *weighted sum* vector by the *corresponding priority* of each criterion
- 6. Calculate the average of the values from the previous step; this value is called ηmax.
- 7. calculate the consistency index (CI)
- 8. Calculate the consistency ratio,

$$CR = CI/RI$$

$$CR = CI/RI = 0004/0.58 = \mathbf{0.006}$$

n	3	4	5	6
RI	0.58	0.9	1.12	1.24

4. Deriving Local Priorities (Preferences) for the Alternatives

- we need to determine the priorities of the alternatives with respect to each of the criteria.

Comparison Question 1: With respect to the COST criterion, which alternative is preferable: Car 1 or Car 2?

Cost	Car 1	Car 2
Car 1	1.000	7.000
Car 2	0.143	1.000
Sum	1.143	8.000

Cost	Car 1	Car 2	Priority
Car 1	0.875	0.875	0.0875
Car 2	0.125	0.125	0.125

Comparison Question 2: With respect to the COMFORT criterion, which alternative is preferable: Car 1 or Car 2?

Comfort	Car 1	Car 2
Car 1	1.000	0.200
Car 2	5.000	1.000
Sum	6.000	1.200

Comfort	Car 1	Car 2	Priority
Car 1	0.167	0.167	0.167
Car 2	0.833	0.833	0.833

Comparison Question 3: With respect to the SAFETY criterion, which alternative is preferable: Car 1 or Car 2?

Safety	Car 1	Car 2
Car 1	1.000	0.111
Car 2	9.000	1.000
Sum	10.000	1.111

Safety	Car 1	Car 2	Priority
Car 1	0.100	0.100	0.100
Car 2	0.900	0.900	0.900

What is the CI of the previous tables ?

Alternatives	Cost	Comfort	Safety
Car 1	0.875	0.167	0.100
Car 2	0.125	0.833	0.900

5. Derive Overall Priorities (Model Synthesis)

Calculate the overall priority (also called final priority)5 for each alternative; that is, priorities that take into account not only our preference of alternatives for each criterion but also the fact that each criterion has a different weight.

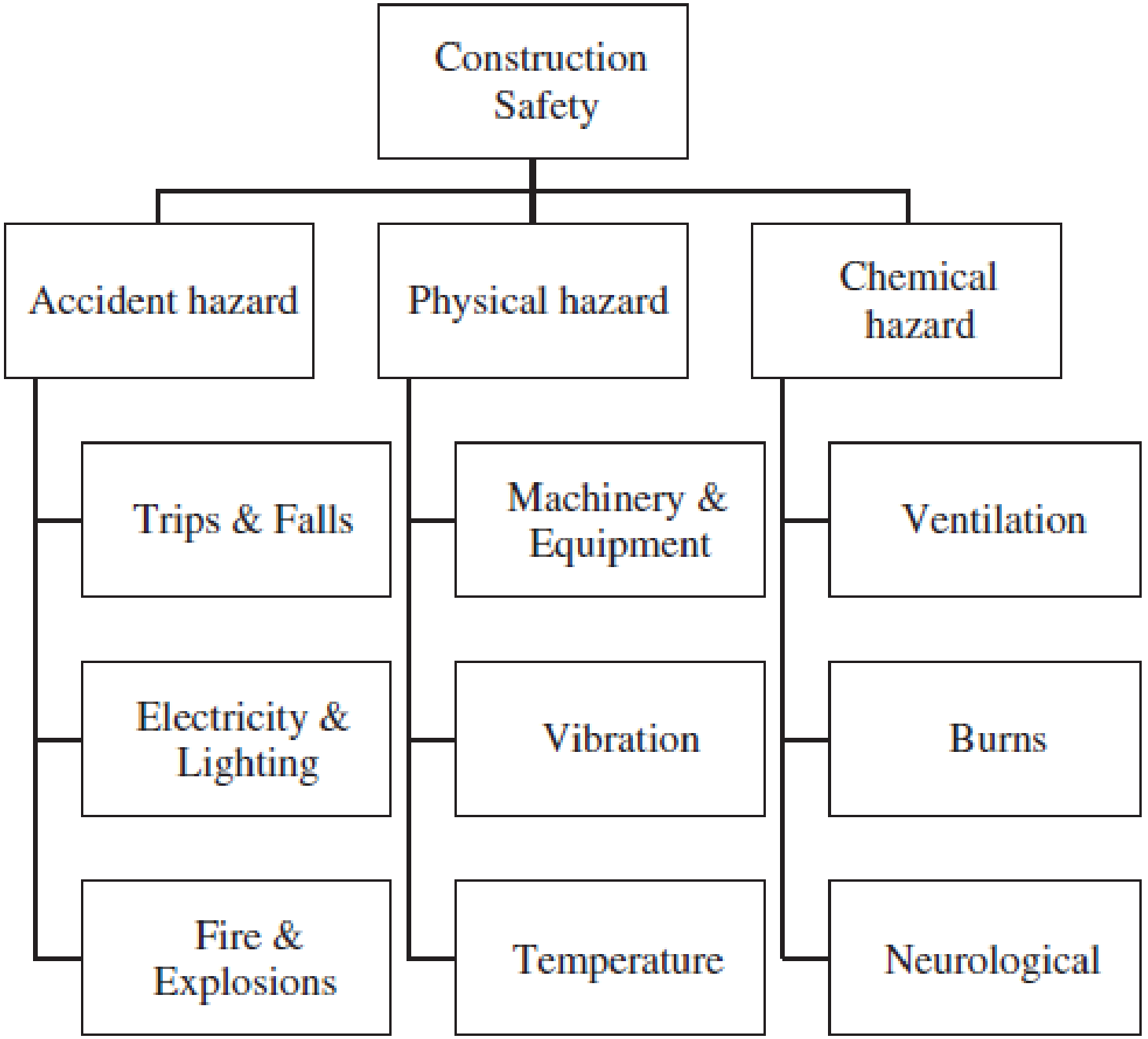
5. Derive Overall Priorities (Model Synthesis)

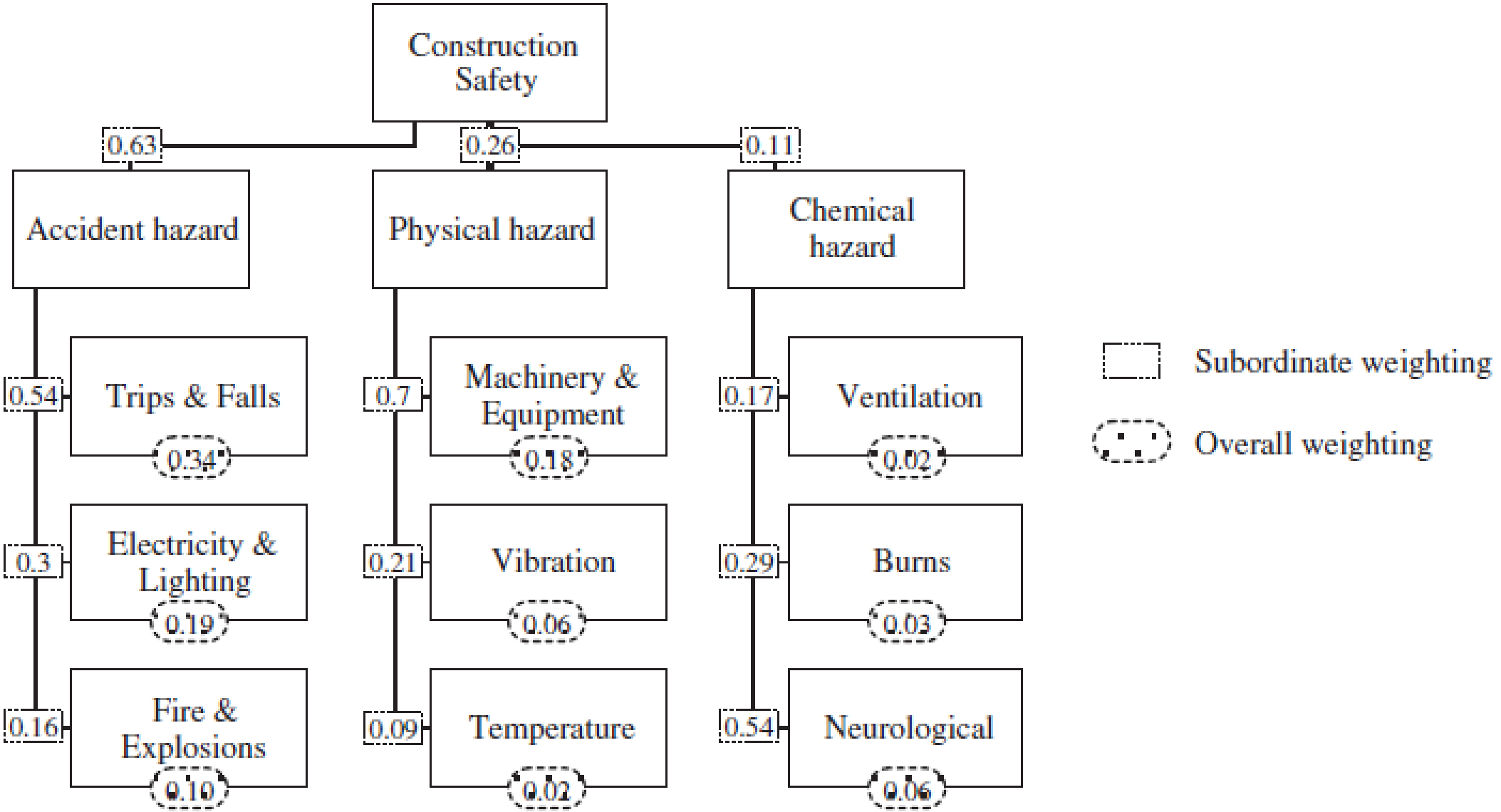
	Cost	Comfort	Safety
Criteria Weights ->	*0.669*	*0.088*	*0.243*
Car 1	0.875	0.167	0.100
Car 2	0.125	0.833	0.900

	Cost	Comfort	Safety	Overall priority
Criteria Weights ->	*0.669*	*0.088*	*0.243*	
Car 1	0.585	0.015	0.024	0.624
Car 2	0.084	0.074	0.219	0.376

Sensitivity Analysis

??????????????????????


Construction Safety
Accident hazard
Physical hazard
Chemical hazard
Trips & Falls
Electricity & Lighting
Fire & Explosions
Machinery & Equipment
Vibration
Temperature
Ventilation
Burns
Neurological


Construction Safety
0.63
0.26
0.11
Accident hazard
Physical hazard
Chemical hazard
0.54
Trips & Falls
0.34
0.3
Electricity & Lighting
0.19
0.16
Fire & Explosions
0.10
0.7
Machinery & Equipment
0.18
0.21
Vibration
0.06
0.09
Temperature
0.02
0.17
Ventilation
0.02
0.29
Burns
0.03
0.54
Neurological
0.06
Subordinate weighting
Overall weighting

AHP APPLICATIONS

- Cost/Benefit Analysis
- Strategic planning
- R&D priority setting and selection
- Technology choice
- Investment priority
- Evaluation of alternatives

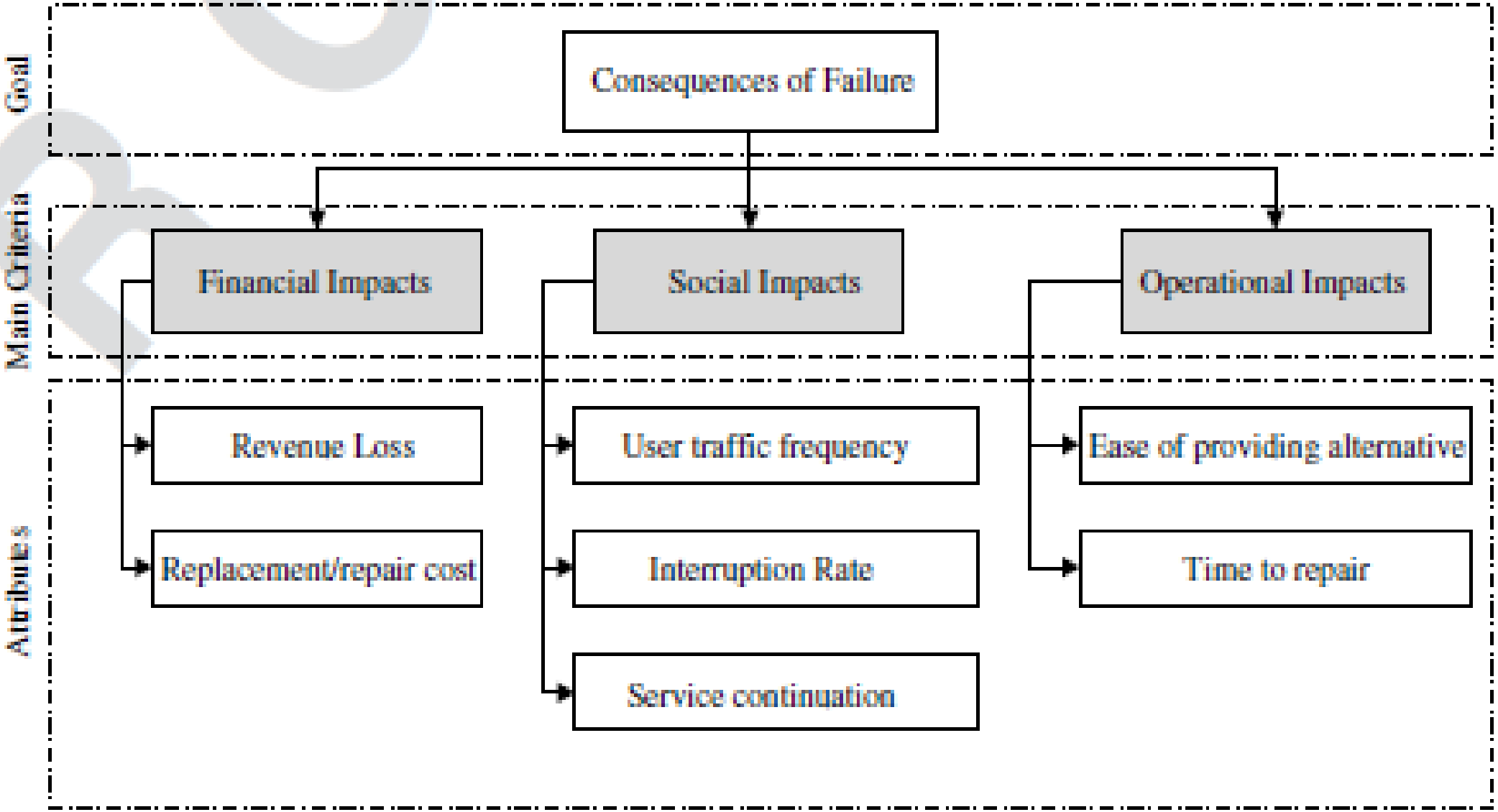


Fig. 4. Consequence of failure attributes.

	Absolute(L)	strong(L)	Strong(L)	Moderate(L)	Equal	Moderate	Strong	strong
Station Characteristics	○	○	○	○	○	○	○	○

B. Main criteria comparison with respect to each other ;

14. With respect to "Station Characteristics", Indicate the relative importance of impacts over eachother *

	9 Absolute(L)	7 Very strong(L)	5 Strong(L)	3 Moderate(L)	1 Equal	3 Moderate	5 Strong	7 Very strong	9 Absol
Station Location	○	○	○	○	○	○	○	○	○

15. With respect to "Station Location", Indicate the relative importance of impacts over eachother *

	9 Absolute(L)	7 Very strong(L)	5 Strong(L)	3 Moderate(L)	1 Equal	3 Moderate	5 Strong	7 Very strong	9 Absolut
Station Size	○	○	○	○	○	○	○	○	○

16. With respect to "Station Nature of use", Indicate the relative importance of impacts over eachother *

	9 Absolute(L)	7 Very strong(L)	5 Strong(L)	3 Moderate(L)	1 Equal	3 Moderate	5 Strong	7 Very strong
Station Characteristics	○	○	○	○	○	○	○	○

C. Sub-criteria comparison with respect to main criteria;

17. With respect to "Station Characteristics", Indicate the relative importance of impacts over eachother *

ASSIGNMENT 1

- In groups of 4-5, perform critique and analysis for ***ANY*** of the following papers (uploaded to course website)
- Steps of a research paper critique process can be reviewed here
  - https://www.ucalgary.ca/ssc/files/ssc/wss_critique_2014.pdf
- Assignment due : 22/10/2019

REFERENCES

- Mu E., Pereyra-Rojas M. (2017) Understanding the Analytic Hierarchy Process. In: Practical Decision Making. Springer Briefs in Operations Research. Springer, Cham. https://doi.org/10.1007/978-3-319-33861-3_2

- Aminbakhsh, S., Gunduz, M., & Sonmez, R. (2013). Safety risk assessment using analytic hierarchy process (AHP) during planning and budgeting of construction projects. *Journal of safety research*, *46*, 99-105.