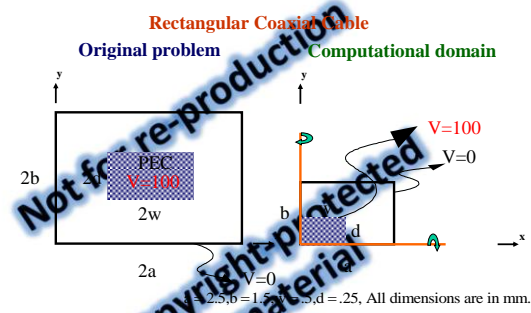


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Efficient Solution of The Quasi-Static Solution of a Rectangular Coaxial Cable Geometry

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Assuming $\Delta x = \Delta y = 0.25$, $a = 2.5, b = 1.5, w = 0.5, d = 0.25$. All dimensions are in mm.

$$V_C = \frac{1}{\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4} \left[V_T \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) + V_L \left(\frac{\epsilon_1 + \epsilon_4}{2} \right) + V_B \left(\frac{\epsilon_4 + \epsilon_3}{2} \right) + V_R \left(\frac{\epsilon_3 + \epsilon_2}{2} \right) \right]$$

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MatLab Program for the Rectangular Coaxial Cable

```

% Rect_coaxial_Potential.m Started on September 6, 2012
% by: Dr. Atef Elsherbeni, atel@uakron.edu
% Computation of the characteristic impedance of a rectangular coaxial cable
% Static solution
% Symmetry along x and y is used, hence, only one quarter only
a = 2.5; % outer length along x
b = 1.5; % outer length along y
w = 0.5; % center conductor length along x
d = 0.25; % center conductor height along y
vcond = 100; % potential of the inner conductor
h = .05; % increment of integer values >>

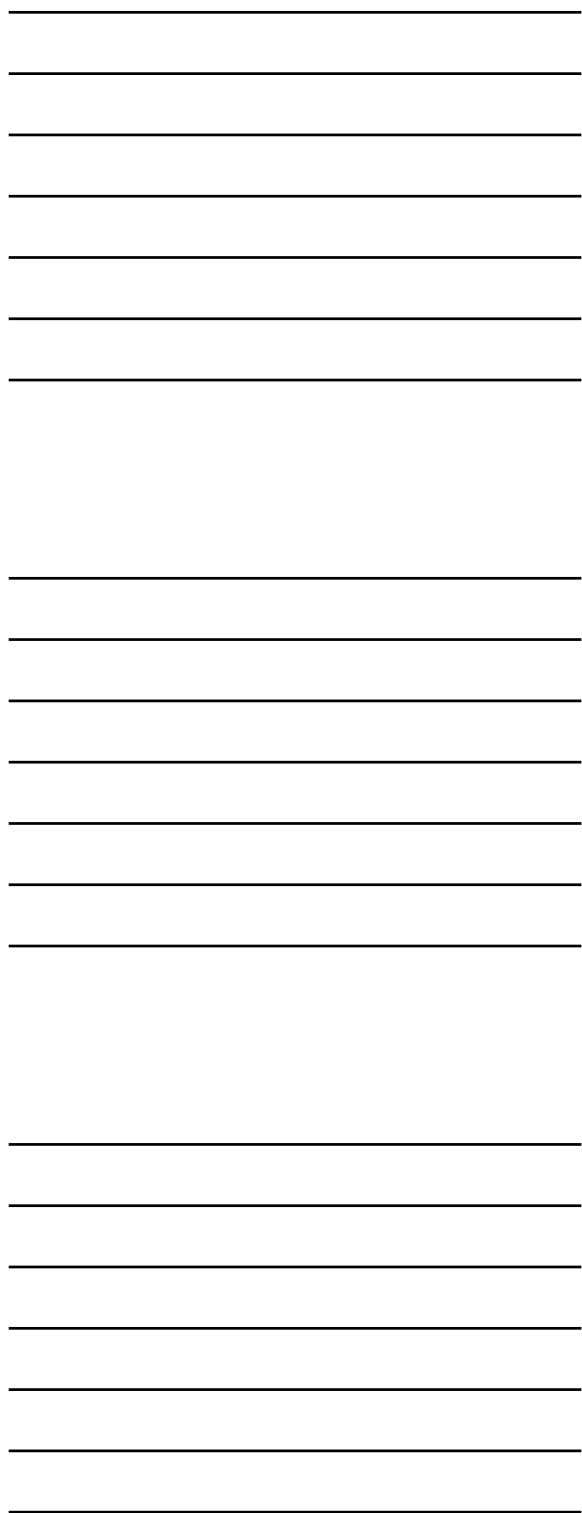
niter = 300; % number of iterations
nx = a/h; ny = b/h; % number of nodes
V(1:nx,1:ny) = 0; % initialization of all nodes to zero

for i = 1:ny
    for j = 1:nx
        if i <= nw & j <= nd
            V(i,j) = vcond; % inner conductor potential
        elseif i == 1 & j > nd
            V(i,j) = .25*(2.*V(i+1,j)+V(i+1,j+1)); % symmetry along y
        elseif j == 1 & i > nd
            V(i,j) = .25*(2.*V(i,j+1)+V(i+1,j+1)); % symmetry along x
        else
            V(i,j) = .25*(V(i+1,j)+V(i-1,j)+V(i,j+1)+V(i,j-1)); % general point
        end
    end
end

surf(V) % plot the potential distribution Note the transpose operation
axis([1 nx 1 ny]); xlabel('x Axis'); ylabel('y Axis');
title('Potential Distribution'); View([-15,35]); % plot the potential distribution

```

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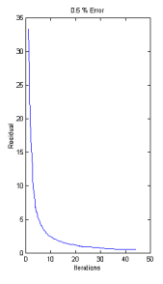


Computation of The Potential With a Specified Accuracy

```

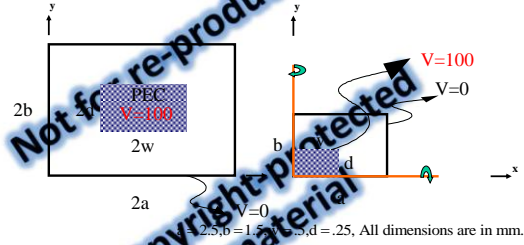
% Rect_coaxial_Pot_Residual.m  September 20, 1999
% by:  Dr. Ataf E. Elsherbeni, atef@uakron.edu
clear all; clf; startops = cputime;
a = 2.5; b = 1.5; % outer dimensions along x and y
w = 0.5; d = 0.25; % center conductor dimensions along x and y
wound = 100; % potential on the conductor
% dimensions of grid
nx = a/hx; ny = b/hy; nw = w/hx; nd = d/hy; % search for integer values >>
% initialize
v(iw,1:ny) = wound; % initialize constant voltage
% percent = 100; % tolerance
percent = 0.100; %min=0.001; % percentage error
for k = 1:1000
    residual = 0; % initialize the residual
    for i = 1:nx
        for j = 1:ny
            % Do nothing fixed potential already defined
            if i <= 1 & j >= ny
                v(i,j) = wound; % symmetry along y
            elseif i == 1 & j <= nd
                v(i,j) = 0; % symmetry along x
            else
                vnew = .25*(v(i+1,j)+v(i-1,j)+2.*v(i,j+1)); % symmetry along y
                vnew = .25*(v(i+1,j)+v(i-1,j)+2.*v(i,j-1)); % symmetry along x
            end
            % absolute error
            f = abs(vnew - v(i,j)); % check for convergence
            if f > residual
                residual = f;
            end
            v(i,j) = vnew;
        end
    end
    % residual and iteration number in array
    line(k) = k;
    resid(k) = residual;
    iter = k;
    if residual < rmin
        break;
    end
end

```



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Rectangular Coaxial Cable Original problem Computational domain



Assuming Δx = Δy = 1, a = 2.5, b = 1.5, w = 0.5, d = 0.25, All dimensions are in mm.

$$V_C = \frac{1}{\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4} \left[V_T \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) + V_L \left(\frac{\epsilon_1 + \epsilon_4}{2} \right) + V_B \left(\frac{\epsilon_4 + \epsilon_3}{2} \right) + V_R \left(\frac{\epsilon_3 + \epsilon_2}{2} \right) \right]$$

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MatLab Program for the Rectangular Coaxial Cable

```

Un-Vectorized code:
for k = 1:1000
    for i = 1:nx
        for j = 1:ny
            if i <= 1 & j >= ny % fixed potential
                v(i,j) = wound;
            elseif i == 1 & j <= nd % symmetry along y
                v(i,j) = 0;
            elseif j == 1 & i >= nw % symmetry along x
                v(i,j) = 0;
            else
                vnew = .25*(v(i+1,j)+v(i-1,j)+2.*v(i,j+1)); % symmetry along y
                vnew = .25*(v(i+1,j)+v(i-1,j)+2.*v(i,j-1)); % symmetry along x
            end
            f = abs(vnew - v(i,j));
            if f > residual
                residual = f;
            end
            v(i,j) = vnew;
        end
    end
    line(k) = k;
    resid(k) = residual;
    iter = k;
    if residual < rmin
        break;
    end
end

```

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Computation of The Potential with a Specified Accuracy - Vectorized Code

```

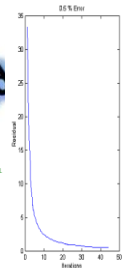
% Rect_coax_Pot_Resid_Opt.m September 20, 1999
% by: Dr. Atef E. Elsharbeni, atef@post.queensu.ca
clear all; clf; close;

DateTime_Start = datestr(now);
start_time=quitime;

a = 2.5; % outer length along x
b = 1.5; % outer length along y
w = 0.5; % center conductor length along x
d = 0.25; % center conductor length along y
vcond = 100; % potential of center conductor
h = -0.5; % segmentation

nx = a/h; ny = b/h; nd = d/h;
% [line1, line2] = 0.0; % initialize all
% [resid] = zeros(nx,ny); % percentage error
% [iter] = zeros(nx,ny); % percentage error

for k = 1:iter
    vnew([nx,ny]) = .25*(v([line1,2ny])+v([line1,2ny])+v([line1,1,1])); % general case
    vnew([nx,1:nd]) = vcond;
    vnew([1,nd:ny]) = .25*(v(2,nd+1:ny)+v(1,nd+1:ny)+v([1,nd+1,1])); % general case
    vnew([1,1:nd]) = .25*(v([2,nd+1,1])+v([2,nd+1,1])+v([1,nd+1,1])); % general case
    [resid] = max(abs(vnew-v));
    if resid < 0.01
        break;
    end
    v = vnew;
    iter(k) = k;
    resid(k) = resid;
end
    
```

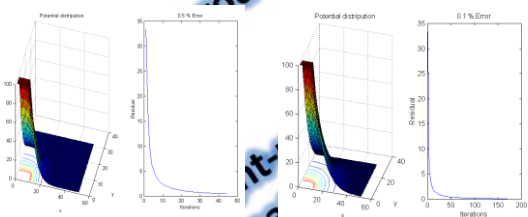


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Number of Iterations Based on Solution Accuracy

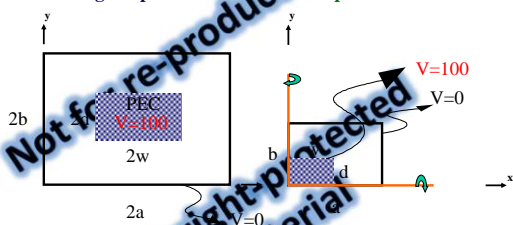


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Rectangular Coaxial Cable



Assuming $\Delta x = 1, \Delta y = 2.5, b = 1.5, w = 0.5, d = .25$, All dimensions are in mm.

$$V_C = \frac{1}{[\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4]} \left[V_T \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) + V_L \left(\frac{\epsilon_1 + \epsilon_4}{2} \right) + V_B \left(\frac{\epsilon_2 + \epsilon_3}{2} \right) + V_R \left(\frac{\epsilon_3 + \epsilon_4}{2} \right) \right]$$

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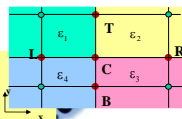
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Coefficients Procedure for Potential Calculation

with $\Delta x = \Delta y = h$

$$V_C = \frac{1}{[\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4]} \left[V_T \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) + V_L \left(\frac{\epsilon_1 + \epsilon_4}{2} \right) + V_B \left(\frac{\epsilon_4 + \epsilon_3}{2} \right) + V_R \left(\frac{\epsilon_2 + \epsilon_3}{2} \right) \right]$$



$$V_C = C_T V_T + C_L V_L + C_B V_B + C_R V_R$$

$$C_T = \frac{\left(\frac{\epsilon_1 + \epsilon_2}{2} \right)}{[\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4]}, \quad C_L = \frac{\left(\frac{\epsilon_1 + \epsilon_4}{2} \right)}{[\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4]}$$

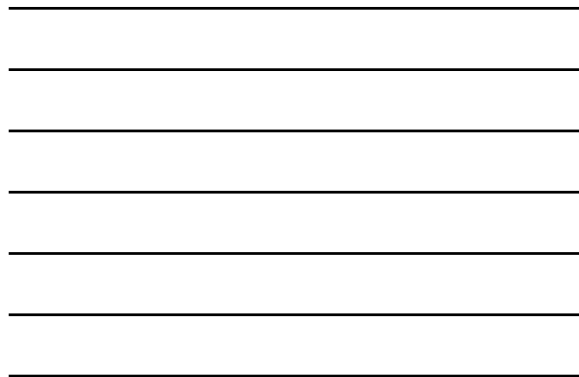
$$C_B = \frac{\left(\frac{\epsilon_4 + \epsilon_3}{2} \right)}{[\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4]}, \quad C_R = \frac{\left(\frac{\epsilon_2 + \epsilon_3}{2} \right)}{[\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4]}$$

Appropriate coefficients should be used for the symmetry and asymmetry conditions.

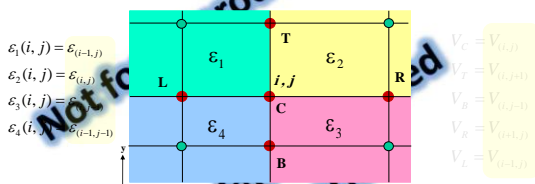
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Re-naming of the Potentials and the Media Properties in Terms of the Node Coordinates



$$V_C = C_T V_T + C_L V_L + C_B V_B + C_R V_R$$

⇓

$$V(i, j) = C_T(i, j)V(i, j+1) + C_L(i, j)V(i-1, j) + C_B(i, j)V(i, j-1) + C_R(i, j)V(i+1, j)$$

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Assignment of Coefficients

$$C_T(i, j) = \frac{\left(\frac{\epsilon_1(i, j) + \epsilon_2(i, j)}{2} \right)}{[\epsilon_1(i, j) + \epsilon_2(i, j) + \epsilon_3(i, j) + \epsilon_4(i, j)]}$$

$$C_L(i, j) = \frac{\left(\frac{\epsilon_1(i, j) + \epsilon_4(i, j)}{2} \right)}{[\epsilon_1(i, j) + \epsilon_2(i, j) + \epsilon_3(i, j) + \epsilon_4(i, j)]}$$

$$C_B(i, j) = \frac{\left(\frac{\epsilon_4(i, j) + \epsilon_3(i, j)}{2} \right)}{[\epsilon_1(i, j) + \epsilon_2(i, j) + \epsilon_3(i, j) + \epsilon_4(i, j)]}$$

$$C_R(i, j) = \frac{\left(\frac{\epsilon_2(i, j) + \epsilon_3(i, j)}{2} \right)}{[\epsilon_1(i, j) + \epsilon_2(i, j) + \epsilon_3(i, j) + \epsilon_4(i, j)]}$$

$$V_T = V_{(i, j+1)}$$

$$V_L = V_{(i-1, j)}$$

$$V_B = V_{(i, j-1)}$$

$$V_R = V_{(i+1, j)}$$

$$V_C = V_{(i, j)}$$

$$V(i, j) = C_T(i, j)V(i, j+1) + C_L(i, j)V(i-1, j) + C_B(i, j)V(i, j-1) + C_R(i, j)V(i+1, j)$$

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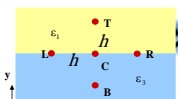
12



Potential at The Interfaces Between Two Media With Uniform Discretization Along x and y Directions

Horizontal boundary

$$V_C = \frac{1}{2[\epsilon_1 \epsilon_2 + \epsilon_1]} \left[V_T \epsilon_1 + V_L \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) + V_B \epsilon_3 + V_R \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) \right]$$

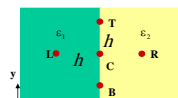


$$C_T(i, j) = \frac{\epsilon_1}{2[\epsilon_1 + \epsilon_1]} \cdot C_T(i, j) = \left(\frac{\epsilon_1}{\epsilon_1 + \epsilon_1} \right)$$

$$C_B(i, j) = \frac{\epsilon_1}{2[\epsilon_1 + \epsilon_2]} \cdot C_B(i, j) = \left(\frac{\epsilon_1}{\epsilon_1 + \epsilon_2} \right)$$

Vertical boundary

$$V_C = \frac{1}{2[\epsilon_1 \epsilon_2 + \epsilon_2]} \left[V_T \epsilon_1 + V_L \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) + V_B \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) + V_R \epsilon_2 \right]$$



$$C_T(i, j) = \frac{\epsilon_1}{2[\epsilon_1 + \epsilon_2]} \cdot C_T(i, j) = \frac{\epsilon_1}{2[\epsilon_1 + \epsilon_2]}$$

$$C_R(i, j) = \frac{\epsilon_2}{2[\epsilon_1 + \epsilon_2]} \cdot C_R(i, j) = \frac{\epsilon_2}{2[\epsilon_1 + \epsilon_2]}$$

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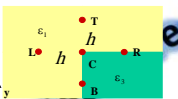
13



Potential at Top Corners With Uniform Discretization Along x and y Directions

Left top corner

$$V_C = \frac{1}{[3\epsilon_1 + \epsilon_2]} \left[V_T \epsilon_1 + V_L \epsilon_1 + V_B \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) + V_R \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) \right]$$

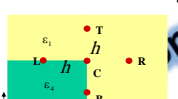


$$C_T(i, j) = \frac{\epsilon_1}{[3\epsilon_1 + \epsilon_2]} \cdot C_T(i, j) = \frac{\epsilon_1}{[3\epsilon_1 + \epsilon_2]}$$

$$C_B(i, j) = \frac{\epsilon_1}{[3\epsilon_1 + \epsilon_2]} \cdot C_B(i, j) = \frac{\epsilon_1}{[3\epsilon_1 + \epsilon_2]}$$

Right top corner

$$V_C = \frac{1}{[3\epsilon_1 + \epsilon_2]} \left[V_T \epsilon_1 + V_L \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) + V_B \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) + V_R \epsilon_1 \right]$$



$$C_T(i, j) = \frac{\epsilon_1}{[3\epsilon_1 + \epsilon_2]} \cdot C_T(i, j) = \frac{\epsilon_1}{[3\epsilon_1 + \epsilon_2]}$$

$$C_R(i, j) = \frac{\epsilon_1}{[3\epsilon_1 + \epsilon_2]} \cdot C_R(i, j) = \frac{\epsilon_1}{[3\epsilon_1 + \epsilon_2]}$$

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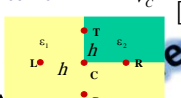
14



Potential at Bottom Corners With Uniform Discretization Along x and y Directions

Right bottom corner

$$V_C = \frac{1}{[3\epsilon_1 + \epsilon_2]} \left[V_T \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) + V_L \epsilon_1 + V_B \epsilon_1 + V_R \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) \right]$$

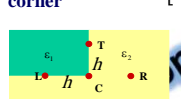


$$C_T(i, j) = \frac{\epsilon_1}{[3\epsilon_1 + \epsilon_2]} \cdot C_T(i, j) = \frac{\epsilon_1}{[3\epsilon_1 + \epsilon_2]}$$

$$C_B(i, j) = \frac{\epsilon_1}{[3\epsilon_1 + \epsilon_2]} \cdot C_B(i, j) = \frac{\epsilon_1}{[3\epsilon_1 + \epsilon_2]}$$

Left bottom corner

$$V_C = \frac{1}{[\epsilon_1 + 3\epsilon_2]} \left[V_T \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) + V_L \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) + V_B \epsilon_2 + V_R \epsilon_2 \right]$$



$$C_T(i, j) = \frac{\epsilon_1}{[3\epsilon_1 + \epsilon_2]} \cdot C_T(i, j) = \frac{\epsilon_1}{[3\epsilon_1 + \epsilon_2]}$$

$$C_B(i, j) = \frac{\epsilon_2}{[\epsilon_1 + 3\epsilon_2]} \cdot C_B(i, j) = \frac{\epsilon_2}{[\epsilon_1 + 3\epsilon_2]}$$

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Potential at point on Lines of Symmetry

Symmetry w.r.t. horizontal line

$$\frac{\partial V}{\partial y} = 0 = \frac{V_T - V_B}{2h} \rightarrow V_T = V_B$$

$$V_C = \frac{1}{4}[2V_B + V_L + V_R]$$

$$V_C = \frac{1}{4}[2V_T + V_L + V_R]$$

$C_T(i, j) = 0$	$C_L(i, j) = \frac{1}{4}$
$C_B(i, j) = \frac{1}{2}$	$C_R(i, j) = \frac{1}{4}$

$C_T(i, j) = \frac{1}{2}$	$C_L(i, j) = \frac{1}{4}$
$C_B(i, j) = 0$	$C_R(i, j) = \frac{1}{4}$

$$V(i, j) = C_T(i, j)V(i, j+1) + C_L(i, j)V(i-1, j) + C_B(i, j)V(i, j-1) + C_R(i, j)V(i+1, j)$$

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Potential at point on Lines of Symmetry

Symmetry w.r.t. vertical line

$$\frac{\partial V}{\partial x} = 0 = \frac{V_R - V_L}{2h} \rightarrow V_R = V_L$$

$$V_C = \frac{1}{4}[V_T + 2V_L + V_B]$$

$$V_C = \frac{1}{4}[V_T + 2V_R + V_B]$$

$C_T(i, j) = \frac{1}{4}$	$C_L(i, j) = \frac{1}{4}$
$C_B(i, j) = \frac{1}{4}$	$C_R(i, j) = 0$

$C_T(i, j) = \frac{1}{4}$	$C_L(i, j) = 0$
$C_B(i, j) = \frac{1}{4}$	$C_R(i, j) = \frac{1}{2}$

$$V(i, j) = C_T(i, j)V(i, j+1) + C_L(i, j)V(i-1, j) + C_B(i, j)V(i, j-1) + C_R(i, j)V(i+1, j)$$

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Potential at point on Lines of Symmetry

Asymmetry w.r.t. horizontal line

$$V_T = -V_B$$

$$V_C = \frac{1}{4}[V_L + V_R]$$

$$V_C = \frac{1}{4}[V_L + V_R]$$

$C_T(i, j) = \frac{1}{4}$	$C_L(i, j) = \frac{1}{4}$
$C_B(i, j) = \frac{1}{4}$	$C_R(i, j) = \frac{1}{4}$

$$V(i, j) = C_T(i, j)V(i, j+1) + C_L(i, j)V(i-1, j) + C_B(i, j)V(i, j-1) + C_R(i, j)V(i+1, j)$$

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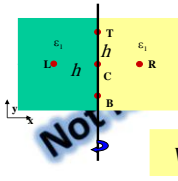
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Potential at point on Lines of Symmetry

Asymmetry w.r.t. vertical line



$$V_R = -V_L$$

$$V_C = \frac{1}{4}[V_T + V_B]$$

$$V_C = \frac{1}{4}[V_T + V_B]$$

$$C_T(i, j) = \frac{1}{4}, C_L(i, j) = 0$$

$$C_B(i, j) = \frac{1}{4}, C_R(i, j) = 0$$

$$V(i, j) = C_T(i, j)V(i, j+1) + C_L(i, j)V(i-1, j) + C_B(i, j)V(i, j-1) + C_R(i, j)V(i+1, j)$$

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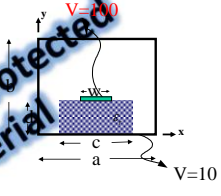
Home Work # 4: Alternative Procedure for Potential Calculation

$$V(i, j) = C_T(i, j)V(i, j+1) + C_L(i, j)V(i-1, j) + C_B(i, j)V(i, j-1) + C_R(i, j)V(i+1, j)$$

Based on the above expression, develop a program to solve for the potential at all the points in the computational domain using the minimum number of code lines for the potential iterative expression (no loops except for the iteration process).

Use the symmetry whenever is possible.

Compare the CPU time based on the coefficient expression procedure with the CPU time if your solution is not based on the coefficient procedure but rather the vectorized code.



a = 10, b = 6, c = 7, d = 2, w = 1.5, $\epsilon_1 = 9$
Thickness of the strip = .05
All dimensions are in mm.

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material

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