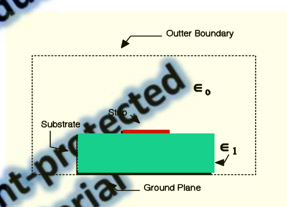
Application of FD to Quasi-Static Solution in Rectangular Coordinates System

The Application of FD Technique to Quasi-Static Solution of a Microstrip Line Geometry



The boundary conditions for a microstrip line are given by:

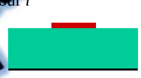
- $V = V_1$ on the strip
- $V = 0$ on the ground plane
- $\oint_S \vec{D} \bullet d\vec{s} = 0$ (Gauss's Law) - dielectric interfaces
- Artificial absorbing boundary condition at the outer boundary of the computational domain.

Potential at Nodes on the Interface between Media Without Free Charges

For a 3D problem $\oint_S \vec{D} \bullet d\vec{s} = Q_{inside}$

For a 2D problem

$\oint_l \vec{D} \bullet \hat{n}\, dl = \rho_{inside}$ where $\hat{n}$ is normal to a closed contour l



Since $\vec{D} = \varepsilon \vec{E} = -\varepsilon \nabla V = -\varepsilon(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y})$

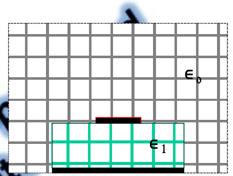
On a dielectric interfaces of a 2D problem without free charges,

we get $-\oint_l \varepsilon(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y}) \bullet \hat{n}\, dl = 0$

The Application of FD Technique to Quasi-Static Solution of a Microstrip Line Geometry

The solution of the potential will be performed at the nodes of the shown **uniform grid** inside the computational domain based on the integral form of Gauss's law

$$\oint_l \varepsilon\left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y}\right) \bullet \hat{n}\, dl = 0$$

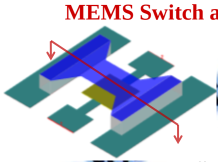


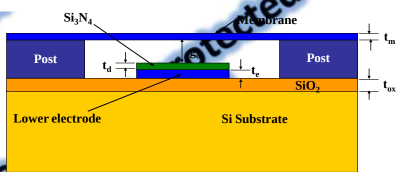
Rather than using Poisson's and/or Laplace's equations

$$\nabla^2 V = {}^{-\rho}\!/_{\varepsilon} \quad \text{and} \quad \nabla^2 V = 0$$

For a more accurate solution, specially for the cases where fine geometrical details are present, it is preferable to use **non-uniform grid**.

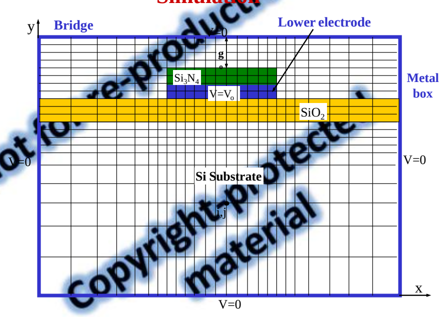
MEMS Switch and 2D Simulation





Ehab K. I. Hamad, Atef Z. Elsherbeni, Amr M. E. Safwat, and Abbas S. Omar, "Two-dimensional Coupled Electrostatic-mechanical Model For RF MEMS Switches," ACES Journal, March 2006.

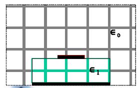
MEMS Switch with Non-Uniform Grid for FD Simulation



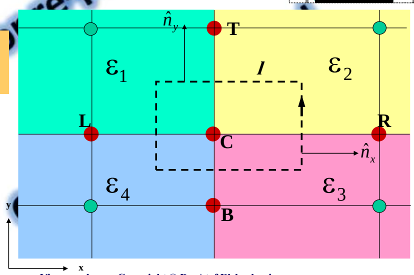
Ehab K. I. Hamad, Atef Z. Elsherbeni, Amr M. E. Safwat, and Abbas S. Omar, "Two-dimensional Coupled Electrostatic-mechanical Model For RF MEMS Switches," ACES Journal, March 2006.

Potential at Nodes at The Intersection Between Four Different Media

$$-\oint_l \varepsilon\left(\frac{\partial V}{\partial x}\hat{x}+\frac{\partial V}{\partial y}\hat{y}\right)\bullet\hat{n}\,dl=0$$



Non-uniform grid in both x and y directions.



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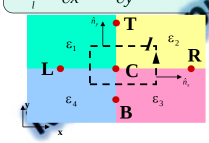
Potential at Nodes at The Interfaces Between Four Different Media

$$-\oint_l \varepsilon\left(\frac{\partial V}{\partial x}\hat{x}+\frac{\partial V}{\partial y}\hat{y}\right)\bullet\hat{n}\,dl=0$$

$$-\left[\begin{array}{l}\int_{right}\varepsilon(y)\frac{\partial V}{\partial x}dy+\int_{top}\varepsilon(x)\frac{\partial V}{\partial y}dx\\ -\int_{left}\varepsilon(y)\frac{\partial V}{\partial x}dy-\int_{bottom}\varepsilon(x)\frac{\partial V}{\partial y}dx\end{array}\right]=0$$

Ignore the negative sign on the left side.



$$\frac{V_R-V_C}{x_R-x_C}\left(\frac{\varepsilon_3(y_C-y_B)/2+\varepsilon_2(y_T-y_C)/2}{(y_T-y_B)/2}\right)\frac{(y_T-y_B)}{2}+\frac{V_T-V_C}{y_T-y_C}\left(\frac{\varepsilon_1(x_C-x_L)/2+\varepsilon_2(x_R-x_C)/2}{(x_R-x_L)/2}\right)\frac{(x_R-x_L)}{2}$$
$$-\frac{V_C-V_L}{x_C-x_L}\left(\frac{\varepsilon_1(y_T-y_C)/2+\varepsilon_4(y_C-y_B)/2}{(y_T-y_B)/2}\right)\frac{(y_T-y_B)}{2}-\frac{V_C-V_B}{y_C-y_B}\left(\frac{\varepsilon_4(x_C-x_L)/2+\varepsilon_3(x_R-x_C)/2}{(x_R-x_L)/2}\right)\frac{(x_R-x_L)}{2}=0$$

Central Difference

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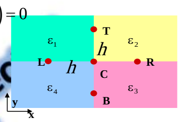
Potential at Nodes at The Interfaces Between Different Media with Uniform Discretization Along *x* and *y* Directions

$$\frac{V_R-V_C}{x_R-x_C}\left(\frac{\varepsilon_3(y_C-y_B)+\varepsilon_2(y_T-y_C)}{2}\right)+\frac{V_T-V_C}{y_T-y_C}\left(\frac{\varepsilon_1(x_C-x_L)+\varepsilon_2(x_R-x_C)}{2}\right)$$
$$-\frac{V_C-V_L}{x_C-x_L}\left(\frac{\varepsilon_1(y_T-y_C)+\varepsilon_4(y_C-y_B)}{2}\right)-\frac{V_C-V_B}{y_C-y_B}\left(\frac{\varepsilon_4(x_C-x_L)+\varepsilon_3(x_R-x_C)}{2}\right)=0$$



Assume $\Delta x=\Delta y=h$

$$(V_R-V_C)\left(\frac{\varepsilon_3+\varepsilon_2}{2}\right)+(V_T-V_C)\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)-(V_C-V_L)\left(\frac{\varepsilon_1+\varepsilon_4}{2}\right)-(V_C-V_B)\left(\frac{\varepsilon_4+\varepsilon_3}{2}\right)=0$$

$$V_C=\frac{1}{[\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4]}\left[V_T\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)+V_L\left(\frac{\varepsilon_1+\varepsilon_4}{2}\right)+V_B\left(\frac{\varepsilon_4+\varepsilon_3}{2}\right)+V_R\left(\frac{\varepsilon_3+\varepsilon_2}{2}\right)\right]$$

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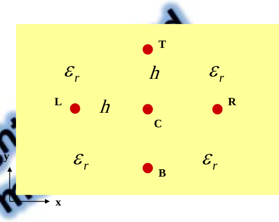
Potential at Nodes in a Uniform Medium With Uniform Discretization Along *x* and *y* Directions

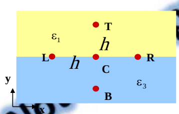
$$V_C = \frac{1}{[\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4]}\left[V_T\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)+V_L\left(\frac{\varepsilon_1+\varepsilon_4}{2}\right)+V_B\left(\frac{\varepsilon_4+\varepsilon_3}{2}\right)+V_R\left(\frac{\varepsilon_3+\varepsilon_2}{2}\right)\right]$$

$$V_C = \frac{1}{4}[V_T+V_L+V_B+V_R]$$



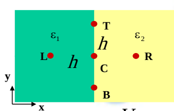
Potential at The Interfaces Between Two Media With Uniform Discretization Along *x* and *y* Directions

General form $$V_C = \frac{1}{[\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4]}\left[V_T\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)+V_L\left(\frac{\varepsilon_1+\varepsilon_4}{2}\right)+V_B\left(\frac{\varepsilon_4+\varepsilon_3}{2}\right)+V_R\left(\frac{\varepsilon_3+\varepsilon_2}{2}\right)\right]$$



Horizontal boundary

$$V_C = \frac{1}{2[\varepsilon_1+\varepsilon_3]}\left[V_T\varepsilon_1+V_L\left(\frac{\varepsilon_1+\varepsilon_3}{2}\right)+V_B\varepsilon_3+V_R\left(\frac{\varepsilon_1+\varepsilon_3}{2}\right)\right]$$



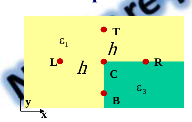
Vertical boundary

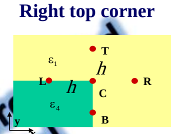
$$V_C = \frac{1}{2[\varepsilon_1+\varepsilon_2]}\left[V_T\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)+V_L\varepsilon_1+V_B\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)+V_R\varepsilon_2\right]$$

Potential at Top Corners With Uniform Discretization Along *x* and *y* Directions

General form $$V_C = \frac{1}{[\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4]}\left[V_T\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right)+V_L\left(\frac{\varepsilon_1+\varepsilon_4}{2}\right)+V_B\left(\frac{\varepsilon_4+\varepsilon_3}{2}\right)+V_R\left(\frac{\varepsilon_3+\varepsilon_2}{2}\right)\right]$$

Left top corner **Right top corner**





$$V_C = \frac{1}{[3\varepsilon_1+\varepsilon_3]}\left[V_T\varepsilon_1+V_L\varepsilon_1+V_B\left(\frac{\varepsilon_1+\varepsilon_3}{2}\right)+V_R\left(\frac{\varepsilon_1+\varepsilon_3}{2}\right)\right]$$

$$V_C = \frac{1}{[3\varepsilon_1+\varepsilon_4]}\left[V_T\varepsilon_1+V_L\left(\frac{\varepsilon_1+\varepsilon_4}{2}\right)+V_B\left(\frac{\varepsilon_1+\varepsilon_4}{2}\right)+V_R\varepsilon_1\right]$$

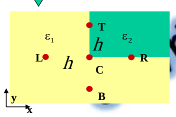
Potential at Bottom Corners With Uniform Discretization Along *x* and *y* Directions

General form

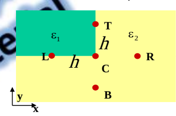
$$V_C = \frac{1}{[\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4]}\left[V_T\left(\tfrac{\varepsilon_1+\varepsilon_2}{2}\right)+V_L\left(\tfrac{\varepsilon_1+\varepsilon_4}{2}\right)+V_B\left(\tfrac{\varepsilon_4+\varepsilon_3}{2}\right)+V_R\left(\tfrac{\varepsilon_3+\varepsilon_2}{2}\right)\right]$$

$$V_C = \frac{1}{[3\varepsilon_1+\varepsilon_2]}\left[V_T\left(\tfrac{\varepsilon_1+\varepsilon_2}{2}\right)+V_L\varepsilon_1+V_B\varepsilon_1+V_R\left(\tfrac{\varepsilon_1+\varepsilon_2}{2}\right)\right]$$

$$V_C = \frac{1}{[\varepsilon_1+3\varepsilon_2]}\left[V_T\left(\tfrac{\varepsilon_1+\varepsilon_2}{2}\right)+V_L\left(\tfrac{\varepsilon_1+\varepsilon_2}{2}\right)+V_B\varepsilon_2+V_R\varepsilon_2\right]$$

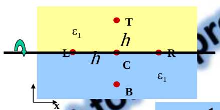


Left bottom corner



Right bottom corner

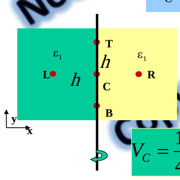
Potential at point on Lines of Symmetry



Symmetry w.r.t. horizontal line

$$\frac{\partial V}{\partial y} = 0 = \frac{V_T - V_B}{2h} \rightarrow V_T = V_B$$

$$V_C = \frac{1}{4}[2V_B + V_L + V_R] \qquad V_C = \frac{1}{4}[2V_T + V_L + V_R]$$

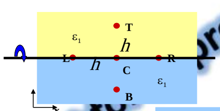


Symmetry w.r.t. vertical line

$$\frac{\partial V}{\partial x} = 0 = \frac{V_R - V_L}{2h} \rightarrow V_R = V_L$$

$$V_C = \frac{1}{4}[V_T + 2V_L + V_B] \qquad V_C = \frac{1}{4}[V_T + 2V_R + V_B]$$

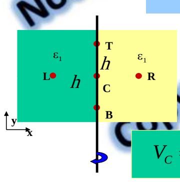
Potential at point on Lines of Anti-Symmetry



Asymmetry w.r.t. horizontal line

$$V_T = -V_B$$

$$V_C = \frac{1}{4}[V_L + V_R] \qquad V_C = \frac{1}{4}[V_L + V_R]$$

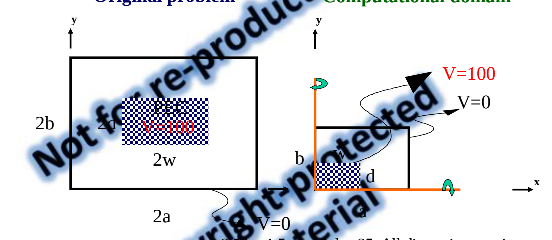


Asymmetry w.r.t. vertical line

$$V_R = -V_L$$

$$V_C = \frac{1}{4}[V_T + V_B] \qquad V_C = \frac{1}{4}[V_T + V_B]$$

Rectangular Coaxial Cable



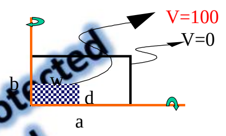
a = 2.5, b = 1.5, w = .5, d = .25, All dimensions are in mm.

Assuming $\Delta x = \Delta y = h$

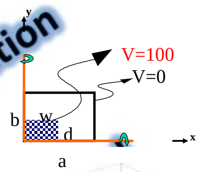
$$V_C = \frac{1}{[\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4]}\left[V_T\left(\frac{\varepsilon_1+\varepsilon_2}{2}\right) + V_L\left(\frac{\varepsilon_1+\varepsilon_4}{2}\right) + V_B\left(\frac{\varepsilon_4+\varepsilon_3}{2}\right) + V_R\left(\frac{\varepsilon_3+\varepsilon_2}{2}\right)\right]$$

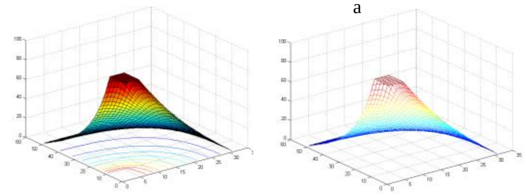
MatLab Program for the Rectangular Coaxial Cable

```
% Rect_coaxial_Potential.m    Started on September 6, 1998
% by: Dr. Atef Z. Elsherbeni, atef@olemiss.edu
% Potential Distribution - rectangular coaxial cable
% Static solution
% Symmetry along x and y is used, hence, computations are for one quarter only
a = 2.5;        % outer lenght along x
b = 1.5;        % outer length along y
w = 0.5;        % center conductor length along x
d = 0.25;       % center conductor length along y
vcond = 100     % potential on the center conductor
h = .05         % incerement of segmentation
                % << watch for integer values >>

niter = 300    % number of iterations
nx = a/h; ny = b/h; nw = w/h; nd = d/h
v(1:nx+1,1:ny+1) = 0.0; % initialization of all nodes to zero

for k = 1:niter
   for i = 1:nx
      for j = 1:ny
         if     i <= nw & j <= nd
               v(i,j) = vcond;                                  % fixed potential
         elseif i == 1 & j > nd
               v(i,j) = .25*(2.*v(i+1,j)+v(i,j+1)+v(i,j-1));   % symmetry along y
         elseif j == 1 & i > nw
               v(i,j) = .25*(v(i+1,j)+v(i-1,j)+2.*v(i,j+1));   % symmetry along x
         else
               v(i,j) = .25*(v(i+1,j)+v(i-1,j)+v(i,j+1)+v(i,j-1)); % general point
         end
      end
   end
end

surfc(v');      % plot the potential distribution Note the transpose operation
axis([1 nx 1 ny]); xlabel('x Axis'); ylabel('y Axis');
title('Potential Distribution');view([-15,35]);     % plot the potential distribution
```



Potential Distribution of a Rectangular Coaxial Cable

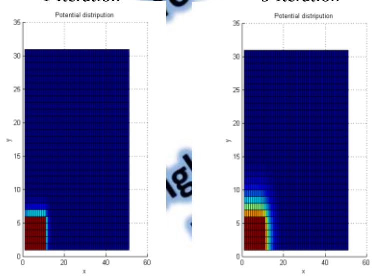




surfc(v) surf(v)

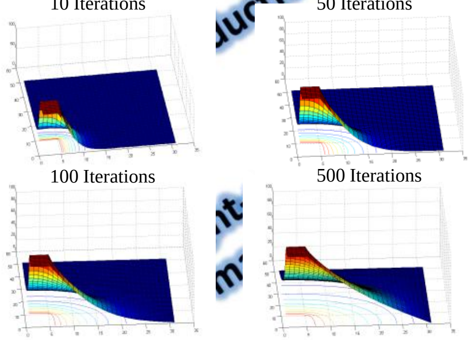
Progress of the Iterative Solution

1 Iteration 9 Iteration



Potential Distribution Based on the Number of Iterations

10 Iterations 50 Iterations
100 Iterations 500 Iterations

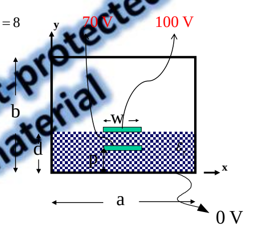


Home Work # 3: Potential Calculation

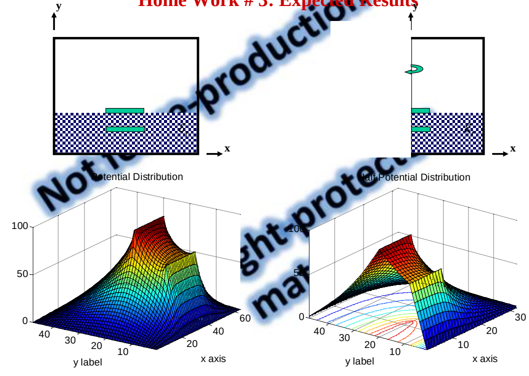
Develop a program to solve for the potential at all points in the shown computational domain. Use the symmetry whenever is possible.

Keep track of the CPU time for each section of the code and for the entire code.

a = 8, b = 6, d = 2.5, p = 1, w = 1.5, $\varepsilon_r = 8$
Thickness of any of the two strips is 0.1
All dimensions are in mm

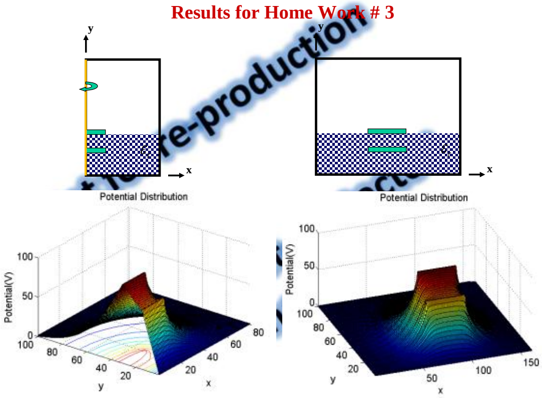


Home Work # 3: Expected Results

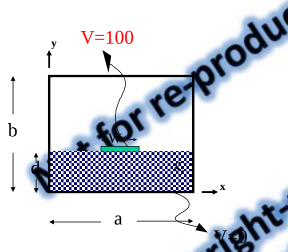


End of Lecture

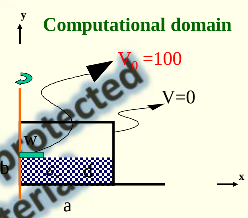
Results for Home Work # 3



Computational Domain for The Microstrip Line Problem



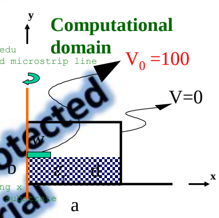
a = 8, b = 5, d = 1.5, w = 2, $\varepsilon_r = 9$
Thickness of the strip =.05
All dimensions are in mm.



a = 4, b = 5, w = 2, d = 1.5, $\varepsilon_r = 9$
Thickness of the strip =.05
All dimensions are in mm.

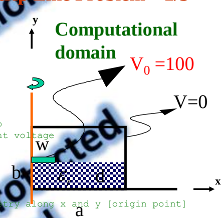
MatLab Program for The Microstrip Line Problem – 1/3


```
% Micro_Strip_Line.m    Started September 23, 1999
% by:                   Dr. Atef Z. Elsherbeni, atef@olemiss.edu
% Computation of the potential distribution for a shielded microstrip line
% Symmetry along y is used
% Last modified August 28, 2007

clear all, clf
eps0 = 8.81*10^-12;    % free space permittivity
c = 2.99*10^8;         % speed of light

% Input Parameters
a =8/2          % half of the outer lenght along x
b = 5;        % half of the outer length along y
w = 2/2;        % half of the center conductor length along x
epsr = 9.0;     % relative permittivity of the dielectric substrate
eps1 = 1;       % top [region 1] air
eps2 = epsr;    % Bottom [ region 2] dielectric
d = 1.5;          % lower Y coordinate of the conducting strip
                % and the top surface of the dielectric substrate
t =  .05;       % thickness of the strip
                % use the increment of x and y equal or multiple integer of t
vcond = 100;    % potential on the center conductor
h = .05;        % incerement of segmentation
Maxiter = 1000; % Maximum number of iterations
% End of Input Parameters
```

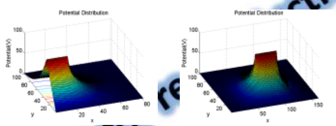


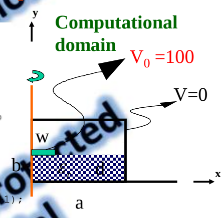
MatLab Program for The Microstrip Line Problem – 2/3


```
eps12 = eps1+eps2;
nx = round(a/h); ny = round(b/h);
nw = round(w/h); nd = round(d/h); nt = round(t/h);
v(1:nx+1,1:ny+1) = 0.0;  % initialize all nodes to zero
v(1:nw,nd:nd+nt-1) = vcond;      % initialize the constant voltage

for k =1:Maxiter
    for i = 1:nx
        for j = 2:ny
            if i ==1 & j ==1
               v(i,j) = .5*(v(i+1,j)+v(i,j+1));  % symmetry along x and y [origin point]
            elseif  i <= nw & nd <= j & j <= nd+nt
               v(i,j) = vcond;                     %  fixed potential
            elseif i==1 &  (j < nd | j > nd + nt)
               v(i,j) = .25*(2.*v(i+1,j)+v(i,j+1)+v(i,j-1));   % symmetry along y
            elseif  i > nw & j == nd
                v(i,j)= (1/(2*(eps1+eps2))) *( ((eps1+eps2)/2).* v(i+1,j) + v(i,j+1)...
                      + eps2.* v(i,j-1)+ ((eps1+eps2)/2).* v(i-1,j)); % potential across 2 media
            else
                v(i,j) = .25*(v(i+1,j)+v(i-1,j)+v(i,j+1)+v(i,j-1)); % general point
            end
        end
    end
end
```



MatLab Program for The Microstrip Line Problem – 3/3






```
vupd = flipud(v);
vt(1:nx+nx+2),1:ny+1) = 0.0;
vt(1:nx+1,1:ny+1) = vupd(1:nx+1,1:ny+1);
vt((nx+2):(nx+nx+2),1:ny+1) = v(1:nx+1,1:ny+1);

figure(1)
surfc(v');              % 3D potential distribution
axis([1 nx+1 1 ny]); xlabel('x');ylabel('y');zlabel('Potential(V)');
title('Potential Distribution');view([-20,45]);

figure(2)
surfc(vt');             % 3D potential distribution
axis([1 nx+nx+2 1 ny]); xlabel('x');ylabel('y');zlabel('Potential(V)');
title('Potential Distribution');view([-20,45]);
```


End of Lecture