Lecture 12

STR403 - Metallic Bridges
"Box-Girder Bridges"

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Lecture 12: Box-Girder Bridges

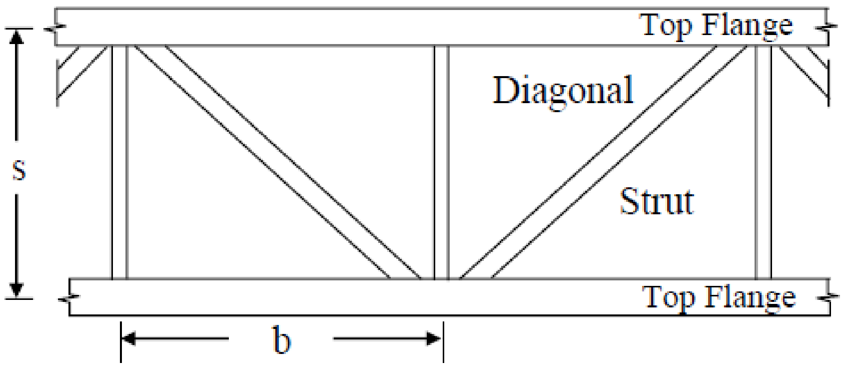
Topics

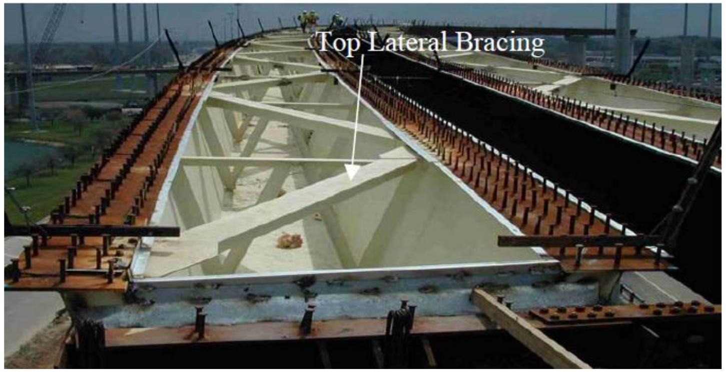
- Behavior of Box-Girder (Continue).
  - Top Flange Truss System

Behavior of Box-Girder

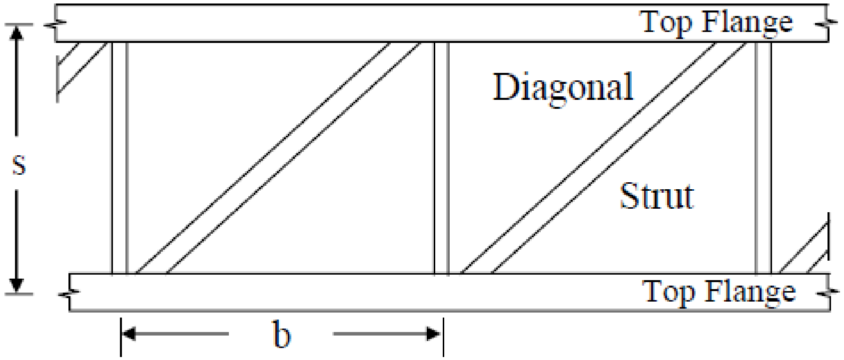
Geometry for Top Flange Lateral Trusses

a) Warren Type Horizontal Top Truss

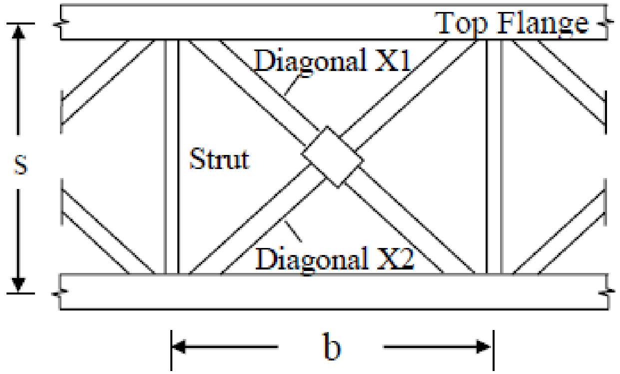




b) Pratt Type Horizontal Top Truss



(c) X-Type Horizontal Top Truss



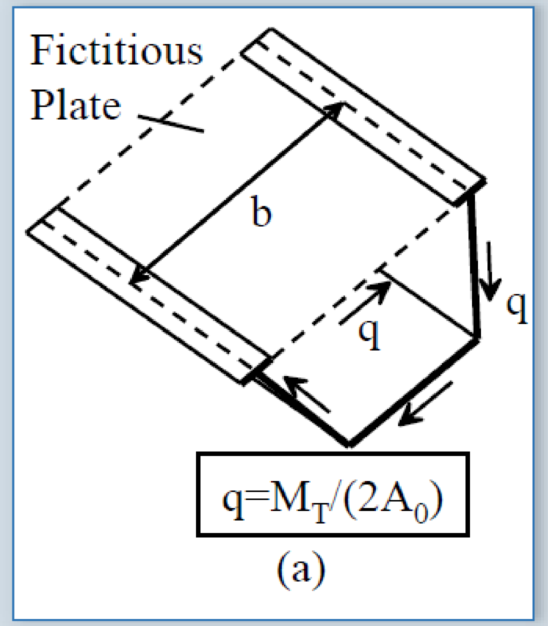
Behavior of Box-Girder

Equivalent Thickness of Top Flange Truss System

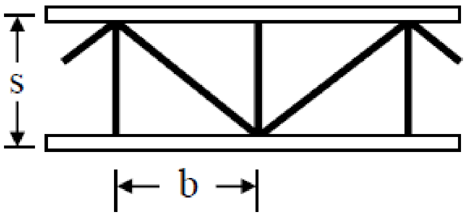
- t^* = equivalent plate thickness;
- E = Modulus of elasticity (29000 ksi for steel);
- G = Shear modulus (11200 ksi for steel);
- b = panel length (spacing between struts);
- s = strut length (width between flanges);
- L_d = diagonal length $\sqrt{\quad}$
- A_d= area of diagonal;
- A_s= area of strut;
- A_f= area of girder top flange (one flange).

EPM

Equivalent Plate Method

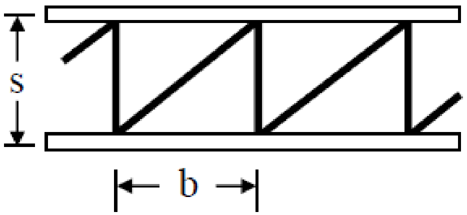


a) Warren Type Horizontal Top Truss



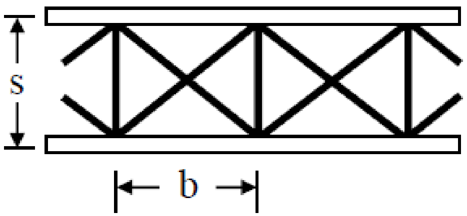
$$t^* = \frac{E}{G}\frac{sb}{\left[\frac{L_d^3}{A_d} + \frac{s^3}{3}\left(\frac{2}{A_f}\right)\right]}$$

b) Pratt Type Horizontal Top Truss



$$t^* = \frac{E}{G}\frac{sb}{\left[\frac{L_d^3}{A_d} + \frac{b^3}{A_s} + \frac{s^3}{12}\left(\frac{2}{A_f}\right)\right]}$$

(c) X-Type Horizontal Top Truss



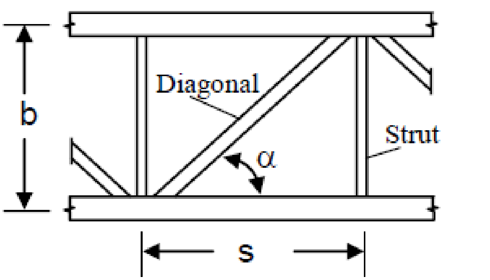
$$t^* = \frac{E}{G}\frac{sb}{\left[\frac{L_d^3}{2A_d} + \frac{s^3}{12}\left(\frac{2}{A_f}\right)\right]}$$

Behavior of Box-Girder

Forces in Top Flange Truss System

1 Bending Induced Forces

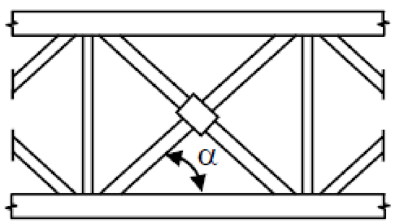
D_{bend} = bending component in diagonal
S_{bend} = bending component in strut
$f_{x\,top}$ = top flange bending stress in panel
s = panel length (spacing between struts)
α = Angle between diagonal and flange
L_d = diagonal length
b = strut length (width between flanges)
A_d, A_s = respective area of diagonal or strut
b_f, t_f = respective width and thickness of girder flange
$f_{L\,bend}$ = lateral bending stress in girder top flange



$$D_{bend} = \frac{f_{xTop}\, s \cos\alpha}{K_1}$$

$$K_1 = \frac{L_d}{A_d} + \frac{b}{A_s}\sin^2\alpha + \frac{s^3}{2b_f^{\,3}t_f}\sin^2\alpha$$

$$S_{bend} = -D_{bend}\sin\alpha$$

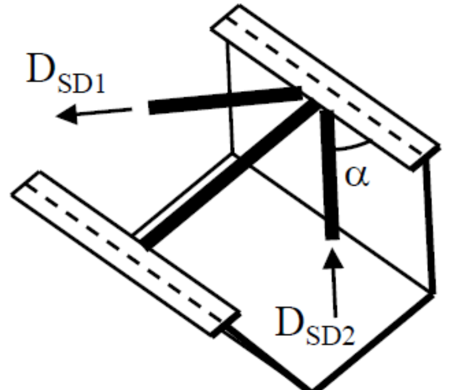


$$D_{bend} = \frac{f_{xTop}\, s \cos\alpha}{K_2}$$

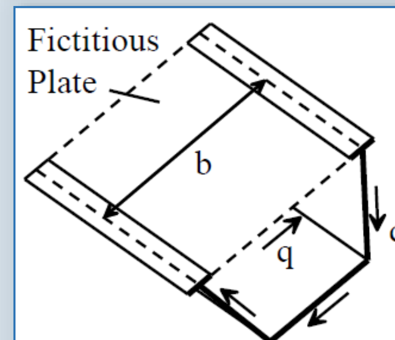
$$K_2 = \frac{L_d}{A_d} + \frac{2b\sin^2\alpha}{A_s}$$

$$S_{bend} = -2D_{bend}\sin\alpha$$

2 Torsion Induced Forces



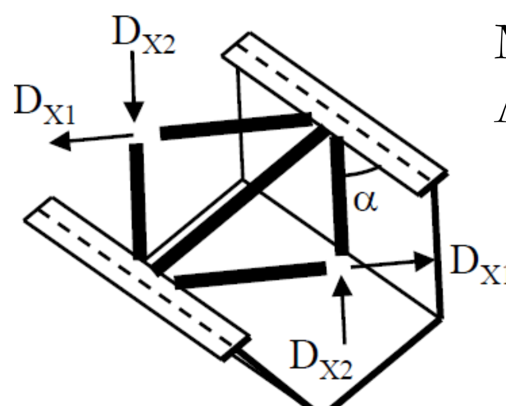
$D_{SD} = \pm\, qb/\sin\alpha$



$q = M_T/(2A_0)$

(a)

q = Shear Flow
M_T = Torsion Moment
$A_0 = \sum b_i\, t_i$

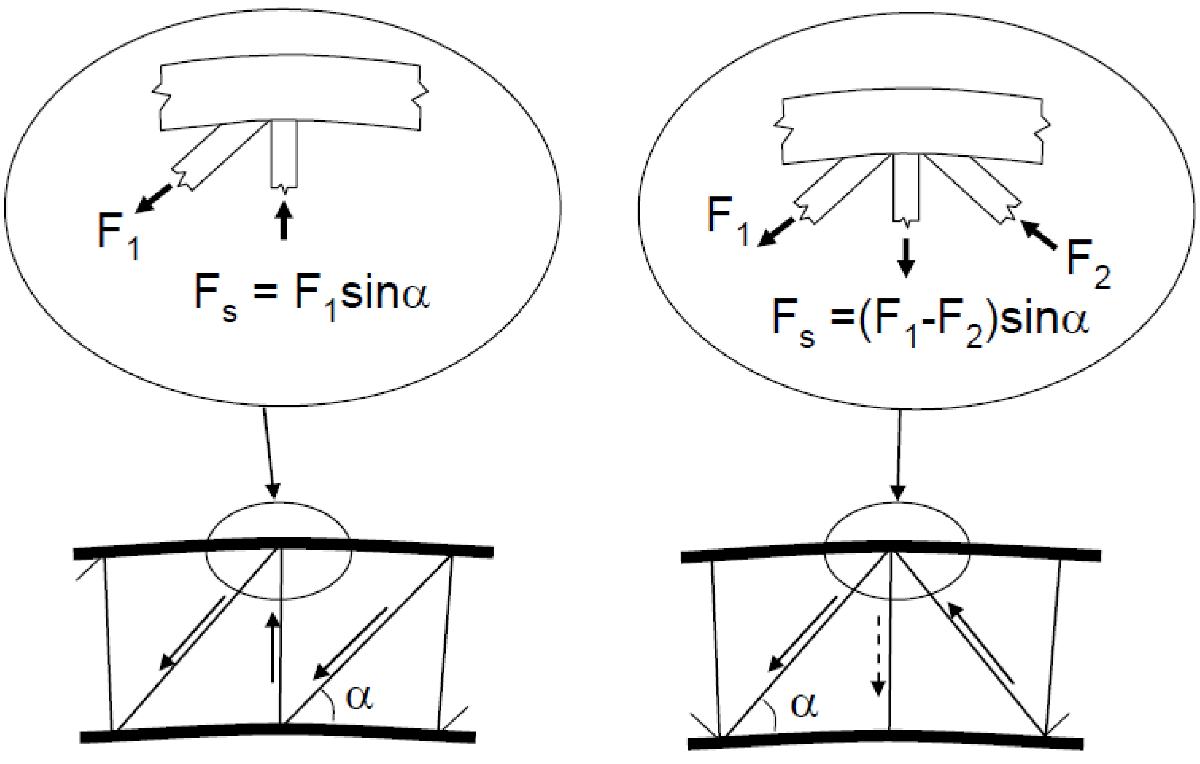


$D_{X1} = -D_{X2} = qb/(2\sin\alpha)$

Superposition:
$D_{bend} + D_{SD}$
$D_{bend} + D_X$

Behavior of Box-Girder

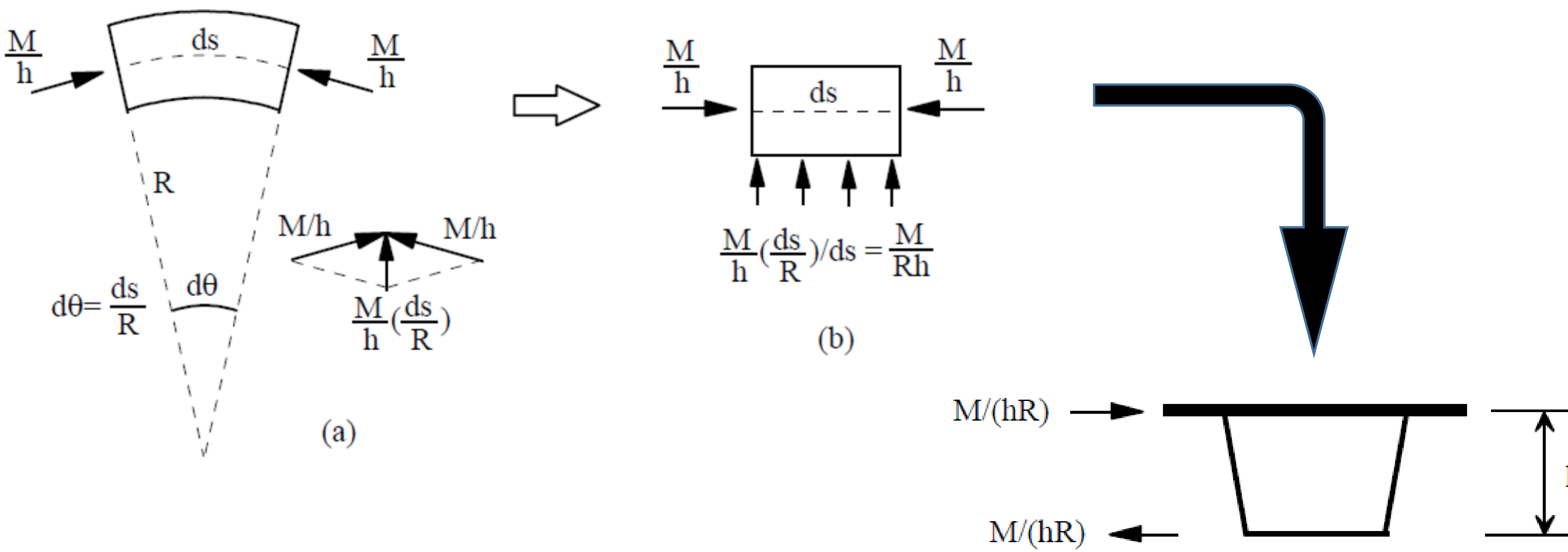
Forces in Top Flange Truss System

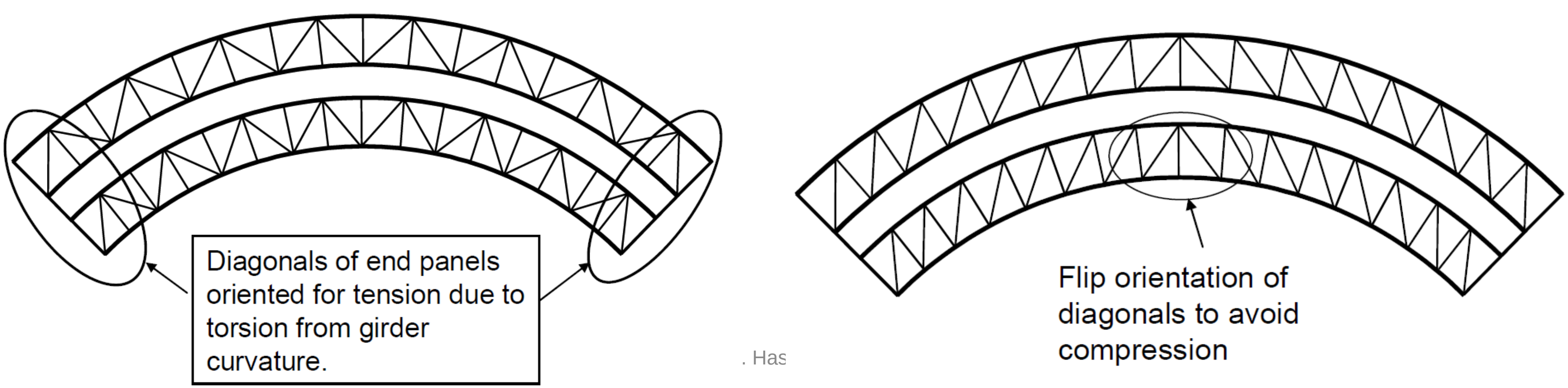


(a) Pratt Truss (b) Warren Truss

Behavior of Box-Girder

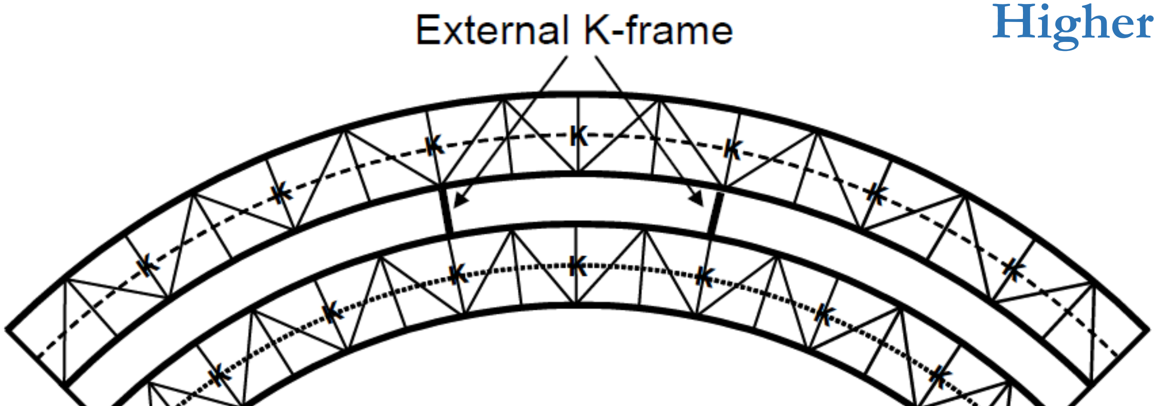
Forces in Top Flange Truss System



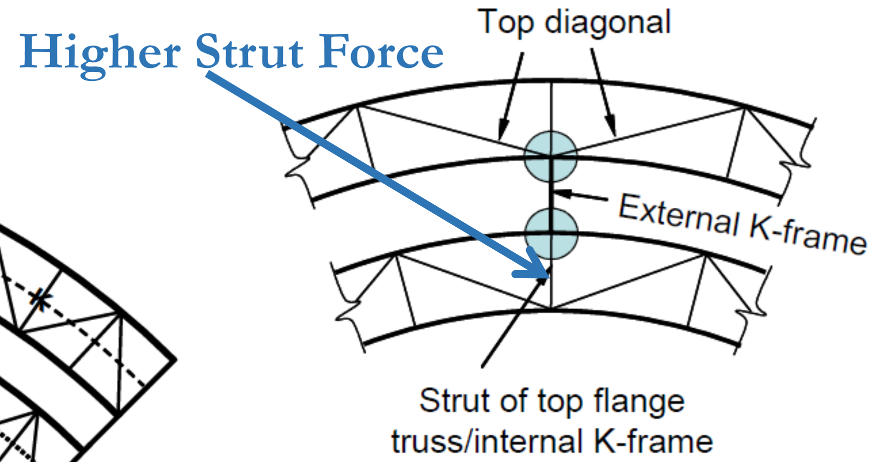


Behavior of Box-Girder

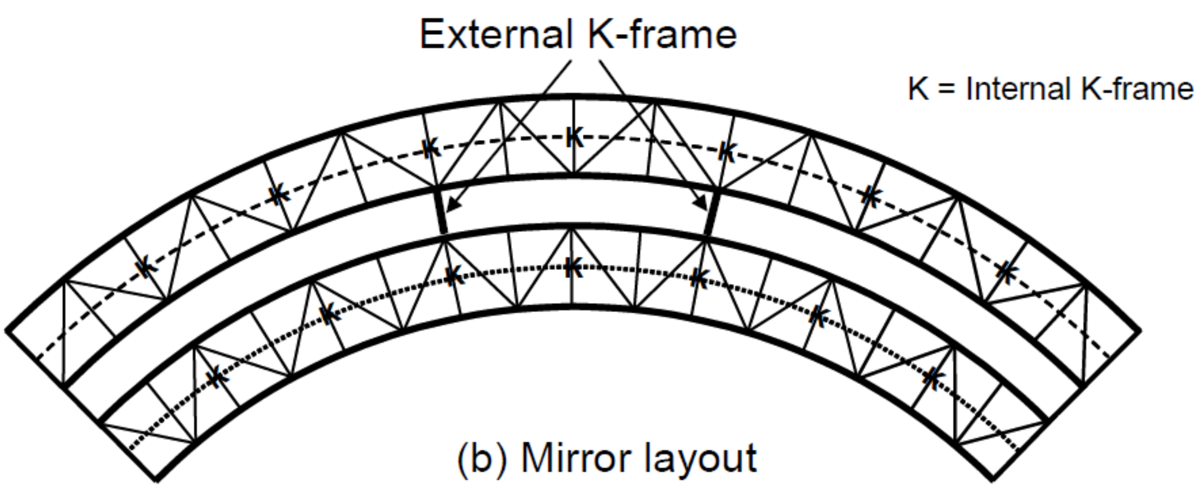
Forces in Top Flange Truss System



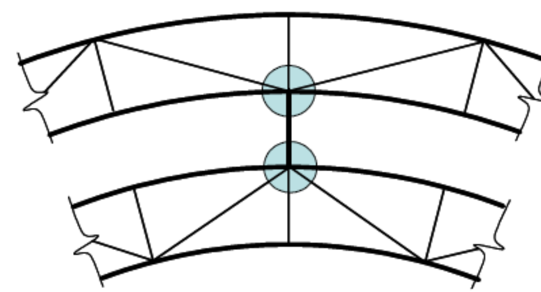
(a) Parallel layout



(a) Parallel layout



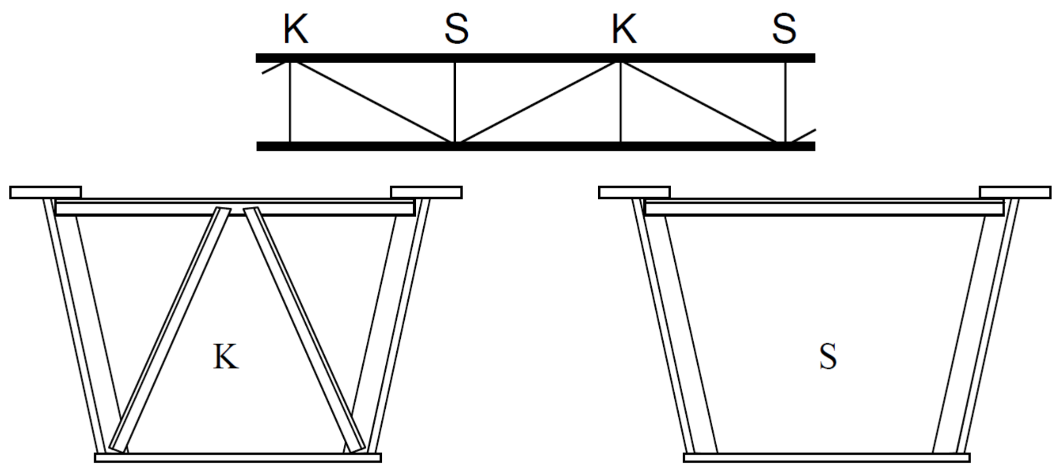
(b) Mirror layout

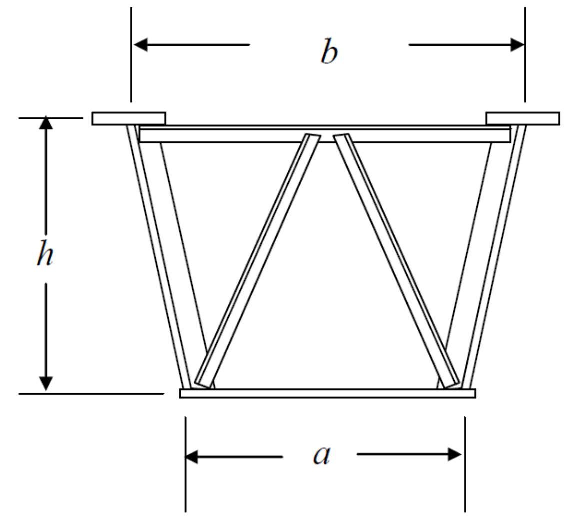

(b) Mirror layout

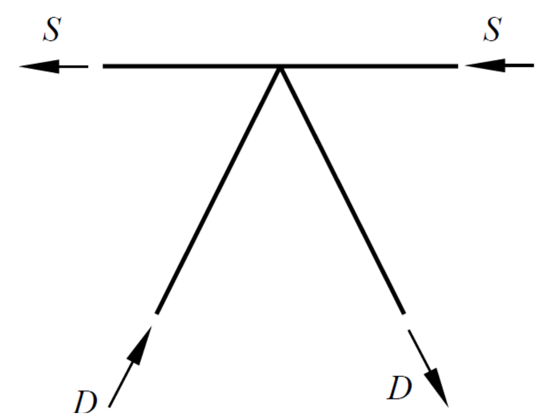
Parallel Versus Mirrored Layout of Top Flange Lateral Truss

Behavior of Box-Girder

Internal K-Frame Forces:







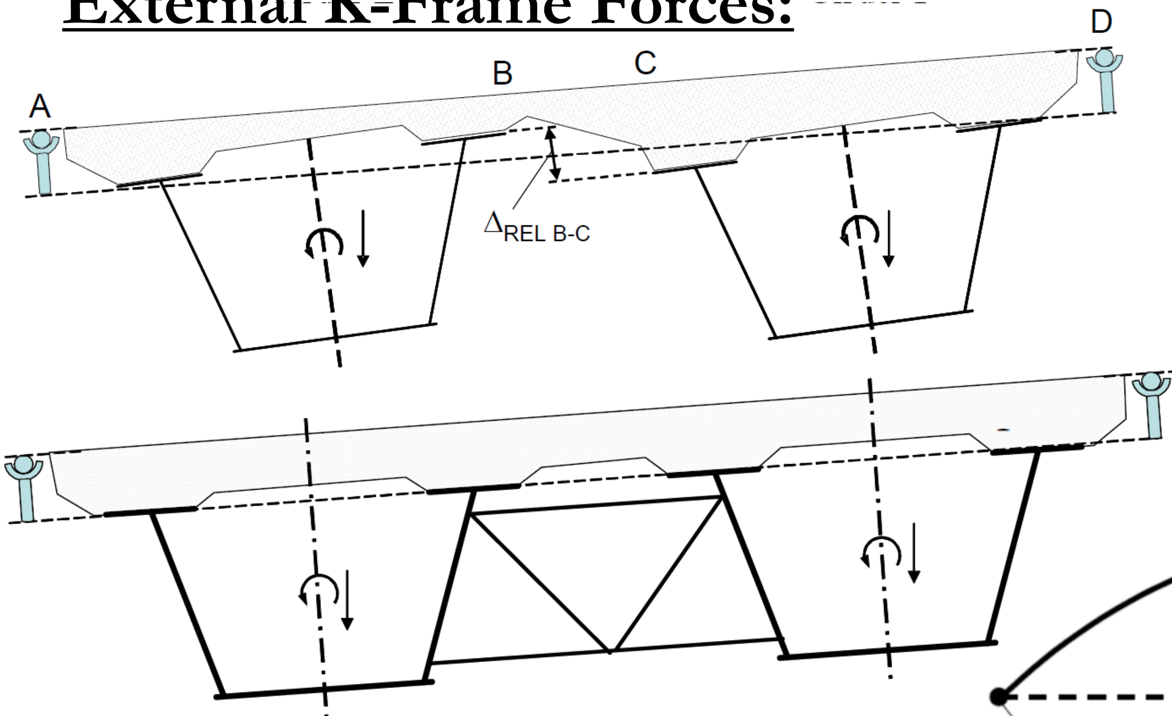
D = distortional induced force in the K-frame diagonal;
S = distortional induced force in the K-frame strut;
s_K = spacing between internal K-frames measured along the girder length;
L_{DK}=length of the K-frame diagonal;
A_0=area enclosed by box=(a+b)/2h;
a and *b* = box girder dimensions as depicted in Figure 4.5a;
e = effective eccentricity of resultant distributed load;
w = distributed load (weight/ft.);
M= box girder bending moment; and
R = radius of horizontal curvature of box.

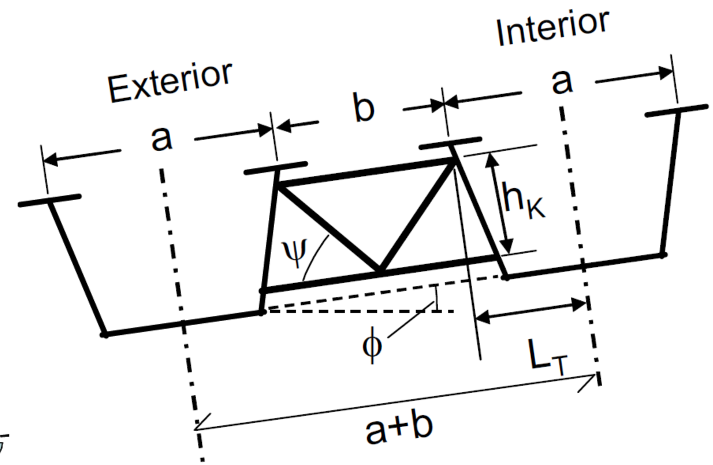
$$D = \pm \frac{s_K L_{DK}}{2A_0}\left(\frac{M}{R} - \frac{a}{b}ew\right)$$

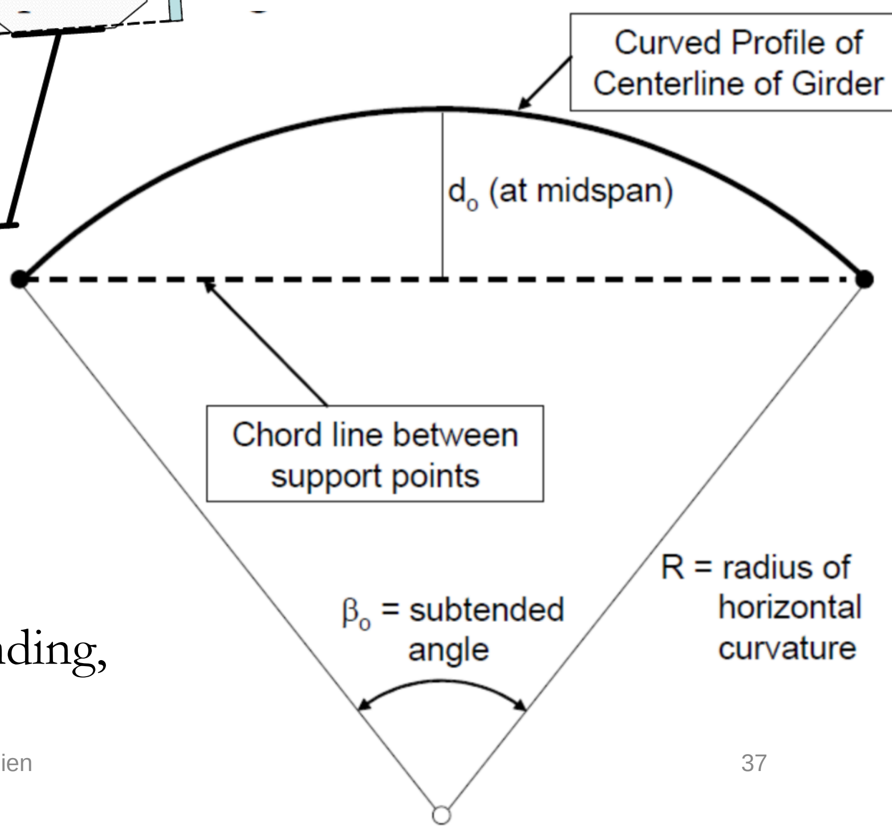
$$S = \pm \frac{s_K a}{4A_0}\left(\frac{a}{b}ew - \frac{M}{R}\right)$$

Behavior of Box-Girder

External K-Frame Forces:







Le = Span of Exterior Girder,

Li = Span of Interior Girder,

βo = Subtended Angle,

R = Horizontal Radius of Curvature,

E = Modulus of Elasticity,

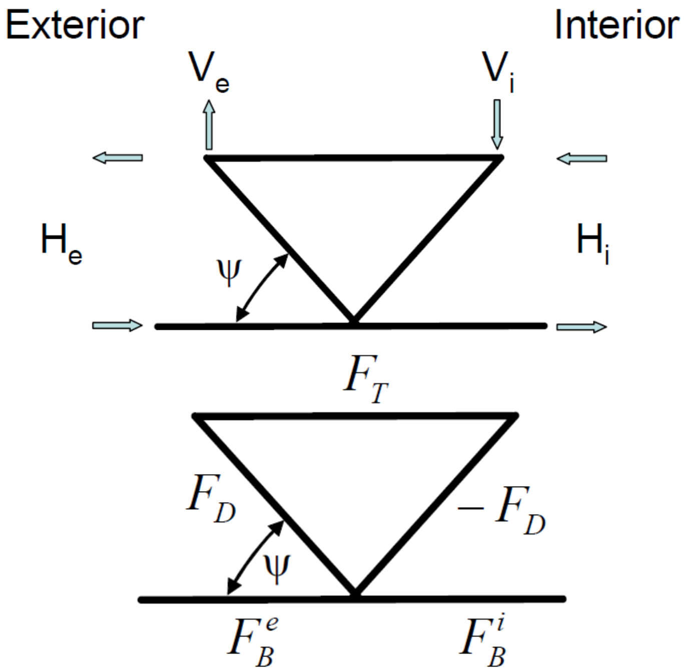
I = Moment of inertia about the axis of bending,

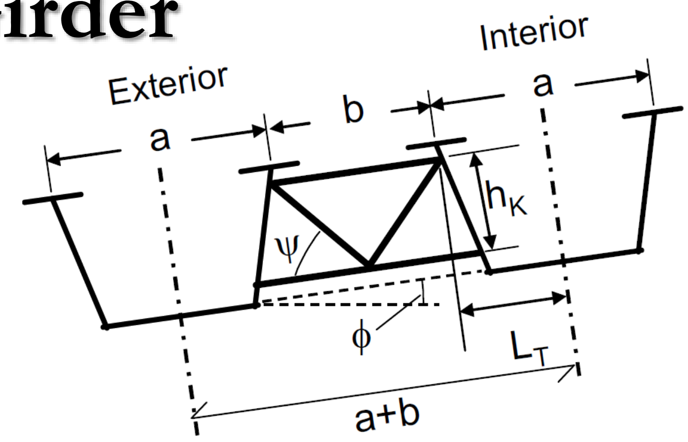
G = Shear Modulus,

J = Torsional Constant = $1/3 \sum b_i t_i^3$

Behavior of Box-Girder

External K-Frame Forces:





$$L_K = h_K \cos\psi + L_T \sin\psi$$

$$K_0 = 1 + \left(1 + \frac{EI}{GJ}\right)\left(1 - \cos\frac{\beta_0}{2}\right)$$

$$K_1 = \frac{L_i + L_e}{a + b}$$

$$K_2 = K_0 K_1 \frac{L_i^3 + L_e^3}{12(EI/GJ)} \sin\psi + 2 L_i L_e L_K$$

$$\Delta_{w,rel} = K_0 \frac{5w}{384EI}\left(L_e^4 - L_i^4\right)$$

$$\phi_{w,\text{int}} = \frac{5wL_i^4}{384EIR_{\text{int}}}\left(1 + \frac{EI}{GJ}\right)$$

$$\phi_{w,ext} = \frac{5wL_e^4}{384EIR_{ext}}\left(1 + \frac{EI}{GJ}\right)$$

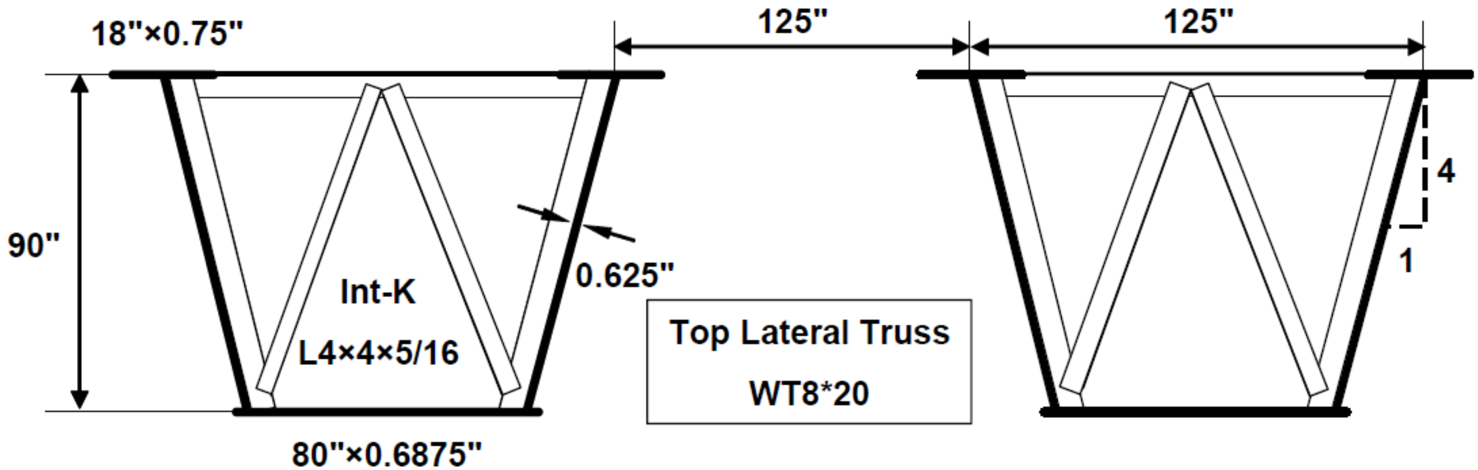
$$F_D = 4GJ \frac{L_i \phi_{w,ext} + L_e \phi_{w,\text{int}} - K_1 \Delta_{w,rel}}{K_2}$$

$$F_T = \frac{4GJ(\phi_{w,ext} - \phi_{w,\text{int}}) - F_D L_K (L_e - L_i)}{h_K (L_i + L_e)}$$

$$F_B = \pm F_D \cos\psi - F_T$$

Behavior of Box-Girder

External K-Frame (Example):



J = 97,000 inch^4 And

I = 234,000 inch^4

$$\beta_0 = \frac{L}{R} = \frac{160}{600} = 0.2667 \quad rad = 15.28°$$

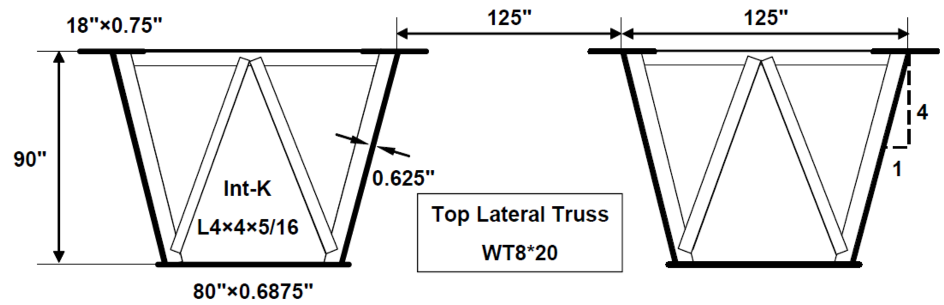
$$\frac{EI}{GJ} = \frac{29000 \times 234000}{112000 \times 97000} = 6.3$$

$$a = b = 10.4'$$

$$L_i = L(\frac{R-b}{R}) = 160 \times (\frac{600-10.4}{600}) = 157.22'$$

Behavior of Box-Girder

External K-Frame (Example):



$$\Delta_{w,\text{int}}(\frac{L}{2}) = \frac{5wL_i^4}{384EI} = \frac{5\times 2\times 157.22^4}{384\times 29000\times 234000/144} = 0.359' = 4.31''$$

$$\phi_{w,\text{int}}(\frac{L}{2}) = \frac{5wL_t^4}{384EIR_{\text{int}}}(1+\frac{EI}{GJ}) = \frac{5\times 2\times 157.22^4}{384\times 29000\times 234000/144\times 589.6}(1+6.3)$$

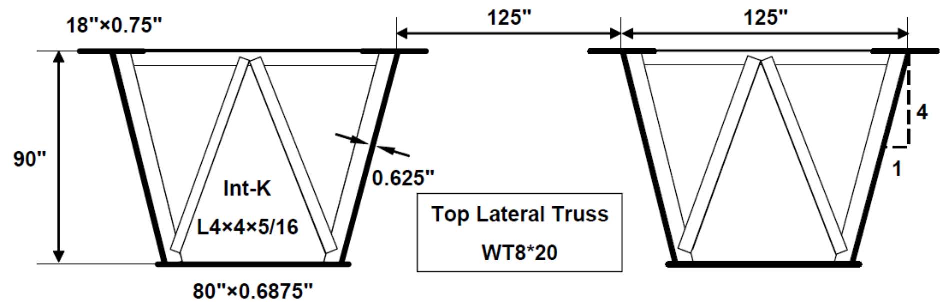
$$= 0.00417 \quad rad$$

$$d_0 = \frac{R_{\text{int}}+R_{ext}}{2}(1-\cos\frac{\beta_0}{2}) = 600'\times(1-\cos 7.64°) = 64''$$

$$L_e = L(\frac{R+b}{R}) = 160\times(\frac{600+10.4}{600}) = 610.4'$$

Behavior of Box-Girder

External K-Frame (Example):



$$\Delta_{w,ext}(\frac{L}{2}) = \frac{5wL_e{}^4}{384EI} = \frac{5\times 2\times 162.78^4}{384\times 29000\times 234000/144} = 0.413' = 4.95''$$

$$\phi_{w,ext}(\frac{L}{2}) = \frac{5wL_e^4}{384EIR_{ext}}(1+\frac{EI}{GJ}) = \frac{5\times 2\times 162.78^4}{384\times 29000\times 234000/144\times 610.4}(1+6.3)$$

$$= 0.00463 \quad rad$$

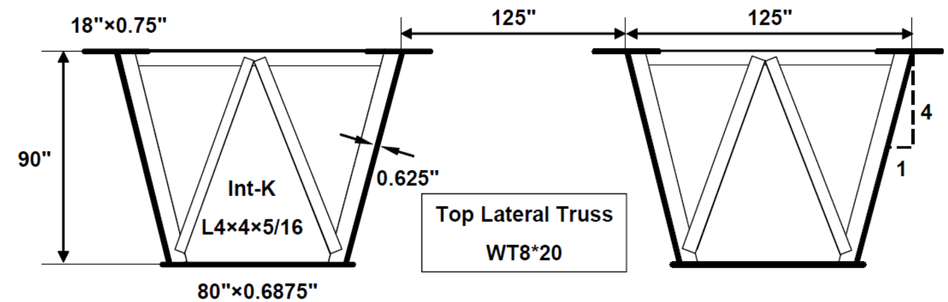
$$h_K = 0.6\times 90'' = 54'' = 4.5'$$

Since the slope of web is ¼, L_T is

$$L_T = \frac{125''}{2} - 0.2\times 90''\times 0.25 = 58'' = 4.83'$$

Behavior of Box-Girder

External K-Frame (Example):



$$\psi = \arctan\left(\frac{h_K}{\frac{(b+a)}{2} - L_T}\right) = \arctan\left(\frac{54}{125-58}\right) = 38.9°$$

$$L_K = h_K \cos\psi + L_T \sin\psi = 4.5 * \cos 38.9° + 4.83 * \sin 38.9° = 6.53'$$

$$K_0 = 1 + \left(1 + \frac{EI}{GJ}\right)\left(1 - \cos\frac{\beta_o}{2}\right) = 1 + (1 + 6.3)\left(1 - \cos\frac{15.27°}{2}\right) = 1.065$$

$$K_1 = \frac{L_i + L_e}{a+b} = 15.36$$

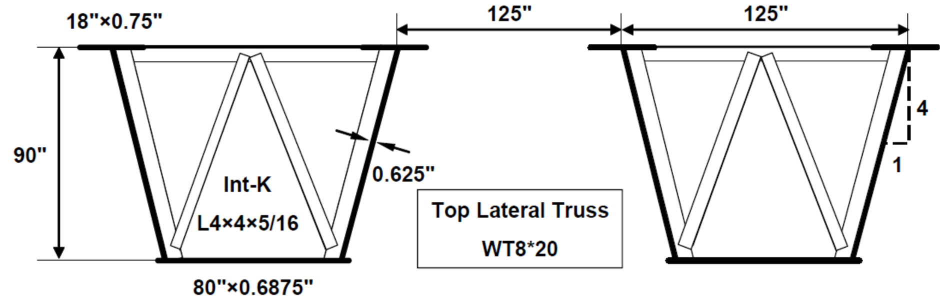
$$K_2 = K_0 K_1 \frac{L_i^3 + L_e^3}{12(EI/GJ)} \sin\psi + 2L_i L_e L_K$$

$$= 1.065 * 15.36 * \frac{157.2^3 + 162.8^3}{12 * 6.3} \sin 38.9° + 2 * 157.2 * 162.8 * 6.53$$

$$= 1{,}448{,}000$$

Behavior of Box-Girder

External K-Frame (Example):



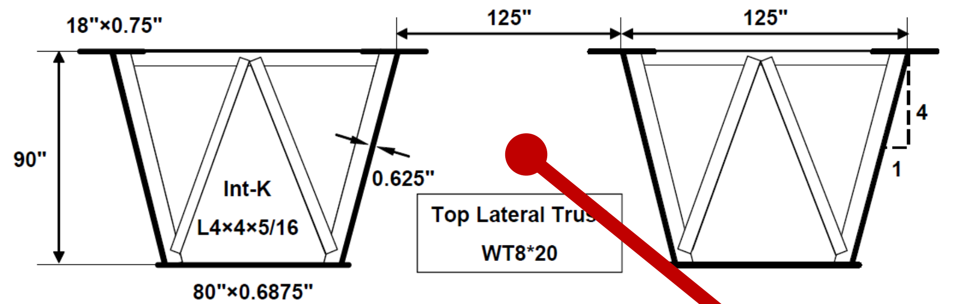
$$\Delta_w = K_0 \frac{5w}{384EI}(L_e^4 - L_i^4) = 1.065 * \frac{5*2.0}{384*29000*234000/144}(162.8^4 - 157.2^4) = 0.6"$$

$$\phi_{w,\text{int}} = \frac{5wL_i^4}{384EIR_{\text{int}}}\left(1+\frac{EI}{GJ}\right) = \frac{5*2.0*157.2^4*7.3}{384*29000*234000/144*589.6} = 0.00418\, rad$$

$$\phi_{w,ext} = \frac{5wL_e^4}{384EIR_{ext}}\left(1+\frac{EI}{GJ}\right) = 0.00463\, rad$$

Behavior of Box-Girder

External K-Frame (Example):



$$F_D = 4GJ\frac{L_i\phi_{w,ext} + L_e\phi_{w,\text{int}} - K_1\Delta_w}{K_2}$$

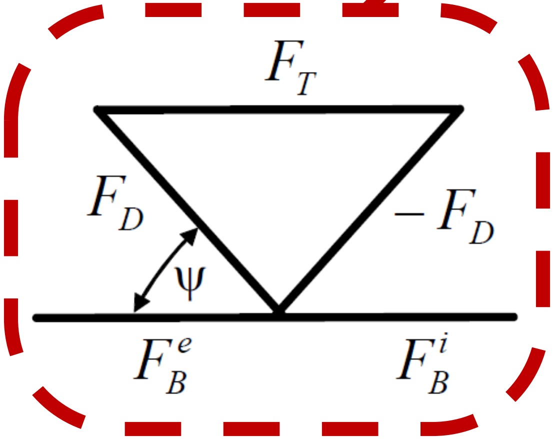
$$= \frac{4*11200*97000}{144} \times \frac{157.2*0.00463 + 162.8*0.00418 - 15.36*0.6/12}{1448000}$$

$$= 13.3\ kips$$

$$F_T = \frac{4GJ(\phi_{w,ext} - \phi_{w,\text{int}}) - F_D L_K (L_e - L_i)}{h'(L_i + L_e)}$$

$$= \frac{4GJ(0.00463 - 0.00418) - 13.3*6.53(162.8 - 157.2)}{4.5(320)}$$

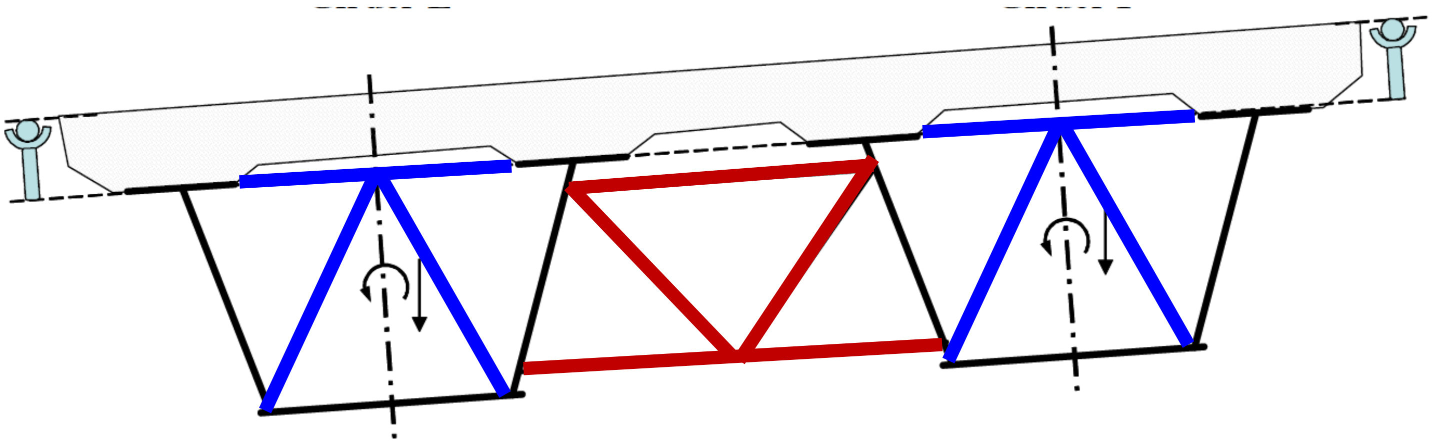
$$= 9.1 kips$$



$$F_B = \pm F_D \cos\psi - F_T = \pm 13.3*\cos 38.9° - 9.1 = \begin{cases} +1.3 \\ -19.4 \quad kips \end{cases}$$

Behavior of Box-Girder

External and Internal K-Frames:



Summary of Today's Topics

- Behavior of Box-Girder (Continue).
  - Top Flange Truss System