METALLIC BRIDGES
STR403

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Previous Lecture

- Bridge Floor Systems.
  - For Roadway Bridges.
  - For Railway Bridges.
- Bridge Loading.
  - For Roadway Bridges.
  - For Railway Bridges.

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This lecture

- Introduction.
- Plate Girder Components.
- Design Considerations:
  - Bucking due to shear
  - Buckling due to bending
  - Stress calculations.
- Inelastic Behavior.
- Effect of Imperfections

INTRODUCTION

Bridges may be classified according to the structural systems of the main girder in the longitudinal direction into several types:

- Beam bridges,
- frame bridges,
- arch bridges,
- cable stayed bridges,
- suspension bridges.

INTRODUCTION

The cross section of the main girder used in any of these bridge types may be a solid web girder or a truss girder depending on the values of the design actions.

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INTRODUCTION

For short and medium spans, solid web girders in the form of I-section or box section are usually used. These girders are usually fabricated from welded plates and thus are called "Plate Girders". Plate girders may be defined as structural members that resist loads primarily in bending and shear. Although shaped similarly to the commonly used hot-rolled steel I-beams, plate girders differ from them in that they are fabricated from plates, and sometimes angles, that are joined together to form I-shapes.

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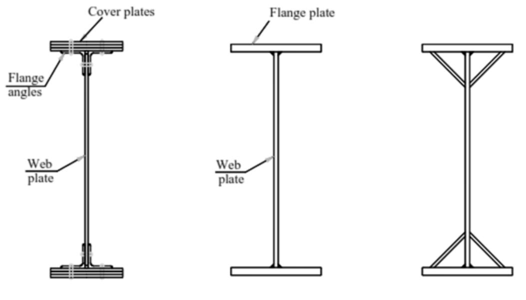
INTRODUCTION

Plate girders are characterized by thin webs, which are usually deeper than those of the deepest available rolled shapes.

Such girders are capable of carrying greater loads over longer spans than is generally possible using standard rolled sections or compound girders. Plate girders may also be used as long-span floor girders in buildings, as crane girders in industrial structures, and as bridge girders in all types of bridges.

CROSS SECTION

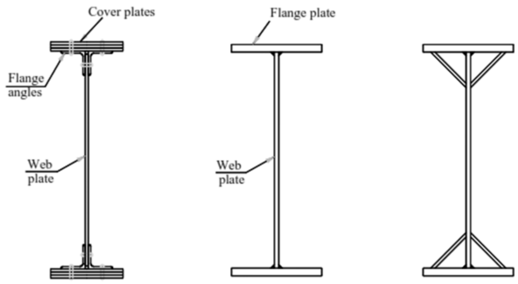
Several cross sections may be used for plate girders:



(a) Riveted (b) Welded (c) Delta

Early plate girders were fabricated by riveting. Their flanges consisted of two angles riveted to the web ends and cover plates riveted to the outstanding legs of the angles.

CROSS SECTION

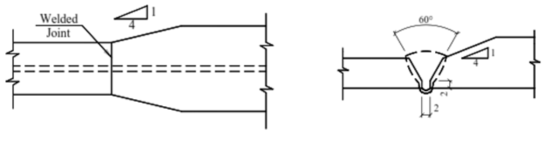


(a) Riveted (b) Welded (c) Delta

Structural welding, which began to be widely used in the 1950s, has significantly simplified the fabrication of plate girders. Modern plate girders are normally fabricated by welding together two flange plates and a web plate. other variations are possible but are less common.

CROSS SECTION

Because a plate girder is fabricated from individual elements, a significant advantage offered by a plate girder is the freedom a designer can have in proportioning the flange and web plates to achieve maximum economy through more efficient arrangement of material than rolled beams. This freedom gives a considerable scope for variation of the cross-section in the longitudinal direction. For example, a designer can reduce the flange width or thickness in a zone of low applied moment.



(a) Width (b) Thickness

CROSS SECTION

Equally, in a zone of high shear, the designer can thicken the web plate. Furthermore, the designer has the freedom to use different grades of steel for different parts of the girder. For example, higher-grade steel might be used for zones of high applied moments while standard grade steel would be used elsewhere. Also, “hybrid girders” with high strength steel in the flange plates and low strength steel in the web offer another possible means of more closely matching resistance to requirements. More unusual variations are adopted in special circumstances, e.g., girders with variable depth (Gama’a Bridge).

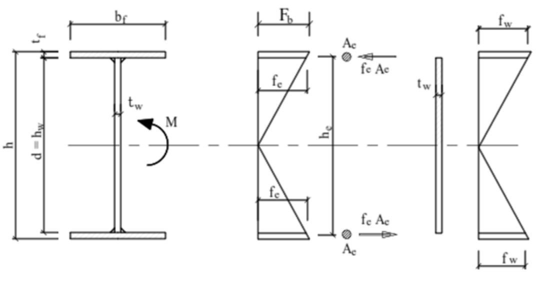
DESIGN CONSIDERATIONS

Any cross-section of a plate girder is normally subjected to a combination of shear force and bending moment. The primary function of the top and bottom flange plates of the girder is to resist the axial compressive and tensile forces arising from the applied bending moment. The primary function of the web plate is to resist the applied shear force. Under static loading, bending and shear strength requirements will normally govern most plate girder design, with serviceability requirements such as deflection or vibration being less critical.

DESIGN CONSIDERATIONS

The first step in the design of plate girder section is to select the value of the web depth. For railway bridges, the girder depth will usually be in the range $L_o/12$ to $L_o/8$, where L_o is the length between points of zero moment.

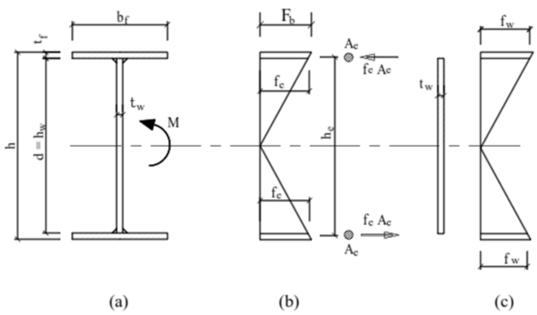
DESIGN CONSIDERATIONS



(a) (b) (c)

However, for plate girder roadway bridges the range may be extended to approximately $L_o/20$ for non-composite plate girders and to $L_o/25$ for composite plate girders. Having selected the web plate depth, the effective flange area to resist the applied moment can be computed from the relation:

DESIGN CONSIDERATIONS



$$M = F_e A_e h_e$$

Where:

F_e = allowable bending stress at flange centroid,

h_e = effective depth for flange,

A_e = equivalent flange area.

DESIGN CONSIDERATIONS

Flange Stress: According to ECP 2001, girders with laterally supported compression flanges can attain their full elastic strength under load, i.e.,

$F_b = 0.64*F_y$ for compact sections and

$F_b = 0.58*F_y$ for non-compact sections. If the compression flange is not supported laterally, then appropriate reduction in the allowable bending stresses shall be applied to account for lateral torsional buckling as set in the Code.

The equivalent flange area A_e is made up of the actual area of one flange, plus the part of the web area that contributes in resisting the applied moment. The moment resistance M_w of the web can be defined by:

DESIGN CONSIDERATIONS

$$M_w = (0.5\ F_w)\ (0.5\ A_w)\ (2h_w/3) = F_w\ h_w\ A_w/6$$

where A_w = area of web, and F_w = maximum bending stress for web. From the above equation it can be seen that one sixth of the total web area can be considered as effective in resisting moment M_w with lever arm h_w and stress F_w. Consequently, the area required for each flange will be: $A_f = A_e - A_w/6$.

Substituting for A_e gives: $A_f = (M / F_b d) - A_w/6$

GIRDER DEPTH

An optimum value of the plate girder depth d which results in a minimum weight girder can be obtained as follows: Express the total girder area as: $A_g = d\ t_w + 2\ A_f$

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DESIGN CONSIDERATIONS

The moment resistance of the girder can be expressed as $M = F_b\ Z_x$

Where Z_x is the section modulus of the girder. Substitution gives: $A_f = Z_x/ d – A_w /6$

Substitution $A_g = 2\ Z_x/ d + 2\ A_w/ 3 = 2\ Z_x/ d + 2\ d\ t_w / 3$
By introducing a web slenderness ratio parameter,

$\beta = d/t_w$, the $A_g = 2\ Z_x / d + 2\ d^2/ 3\ \beta$

A_g is minimum when $\partial A_g / \partial d = 0$ which gives:

$$d^3 = 1.5\ \beta\ Z_x$$

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DESIGN CONSIDERATIONS

Substituting $Z_x = M / F_b$ gives:

$$d = \sqrt[3]{1.5\,\beta\, M / F} \;...$$

The value of β will normally lie in the range 100 to 150. With M expressed in meter-ton units and F in t/cm^2 units, the above equation gives the optimum girder depth in meters as:

$$d = (0.25 \sim 0.3)\sqrt[3]{M / F}$$

For steel S355 with $F_b = 0.58\ F_y$ this equation gives:

$$d = (0.2 \sim 0.24)\sqrt[3]{M}$$

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PLATE GIRDER BUCKLING

For efficient design it is usual to choose a relatively deep girder, thus minimizing the required area of flanges for a given applied moment. This obviously results in a deep web whose thickness t_w is chosen equal to the minimum required to carry the applied shear. Such a web may be quite slender, i.e. has a high d/t_w ratio, and may be subjected to buckling which reduces the section strength. A similar conflict may exist for the flange plate proportions. The desire to increase weak axis inertia encourages wide, thin flanges, i.e. flange with a high b/t_f ratio. Such flanges may also be subjected to local buckling.

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PLATE GIRDER BUCKLING

Design of plate girders therefore differs from that of rolled sections because the latter generally have thicker web and flange plates and thus are not subjected to buckling effects. In contrast, the freedom afforded in material selection in plate girder design makes buckling a controlling design criterion. Thus, in designing a plate girder it is necessary to evaluate the buckling resistance of flange plates in compression and of web plates in shear and bending. In most cases various forms of buckling must be taken into account. There are different buckling problems associated with plate girder design:

PLATE GIRDER BUCKLING

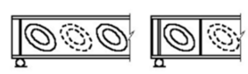
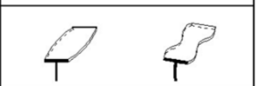
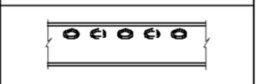
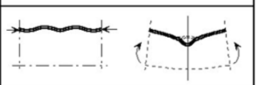
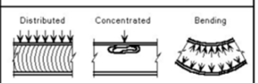
Buckling type	Illustration
a- Shear buckling of web	
b- Lateral-torsional buckling of girder	
c- Local buckling of compression flange	
d- Compression buckling of web	
e- Flange induced buckling of the web	
f- Local buckling of web (due to vertical load)	

PLATE GIRDER BUCKLING

a) Shear Buckling of the Web Plate

If the web width-to-thickness ratio d/t_w exceeds a limiting value, the web will buckle in shear before it reaches its full shear capacity. Diagonal buckles resulting from the diagonal compression associated with the web shear will form. This local buckling reduces the girder shear strength.

PLATE GIRDER BUCKLING

b) Lateral Torsional Buckling of girder

If the compression flange is not supported laterally the girder is subjected to lateral torsional buckling which reduces the allowable bending stresses.

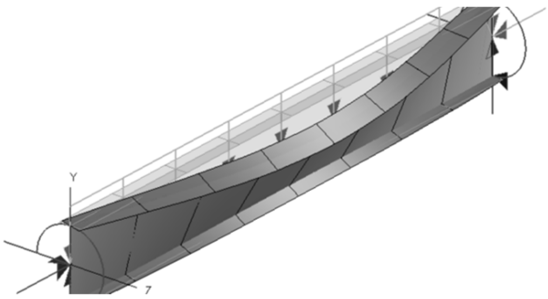


PLATE GIRDER BUCKLING

c) Local Buckling of the Compression Flange

If the compression flange width-to-thickness ratio exceeds a limiting value, it will buckle before it reaches its full compressive strength. This local buckling will reduce the girder's load carrying resistance.

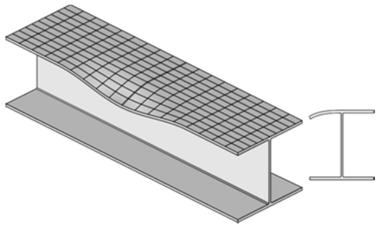


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PLATE GIRDER BUCKLING

d) Compression Buckling of the Web Plate

If the web width-to-thickness ratio d/t_w exceeds a limiting value, the upper part of the web will buckle due to bending compression. Consequently, the moment resistance of the cross section is reduced.

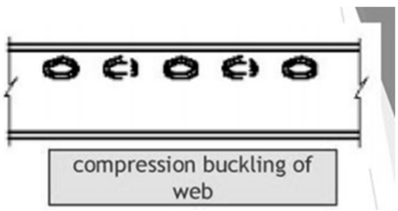

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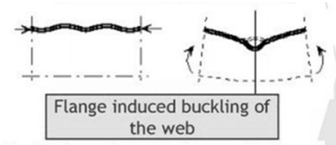
PLATE GIRDER BUCKLING

e) Flange Induced Buckling of the Web Plate

If particularly slender webs are used, the compression flange may not receive enough support to prevent it from buckling vertically rather like an isolated strut buckling about its minor axis. This possibility may be eliminated by placing a suitable limit on d/t_w.



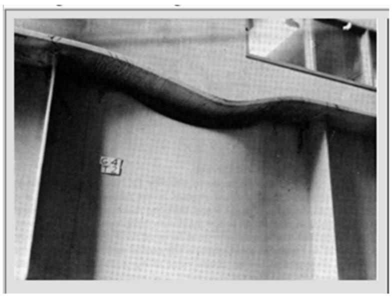


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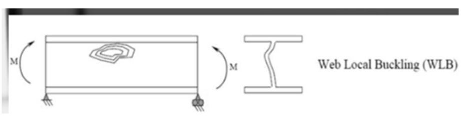
PLATE GIRDER BUCKLING

f) Local Buckling of the Web Plate

Vertical loads may cause buckling of the web in the region directly under the load. This buckling form is known as web crippling. The level of loading that may safely be carried before this happens will depend upon the exact way in which the load is transmitted to the web and the web proportions.



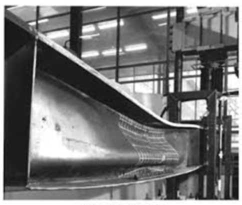


Figure 2. Vertical web buckling.

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PLATE GIRDER BUCKLING

In order to study the effect of local buckling on the strength of the cross-section, knowledge of the theory of buckling of rectangular plates is essential. Flanges can be modeled as long plates under uniform compression with one long edge assumed simply supported and the other long edge free. Webs can be modeled as long plates with the two long edges as simply supported. The compression on the plate edge may be uniform, as in the girder flange, or non-uniform, as in the girder web. In addition, the web plate may be subjected to shear stresses.

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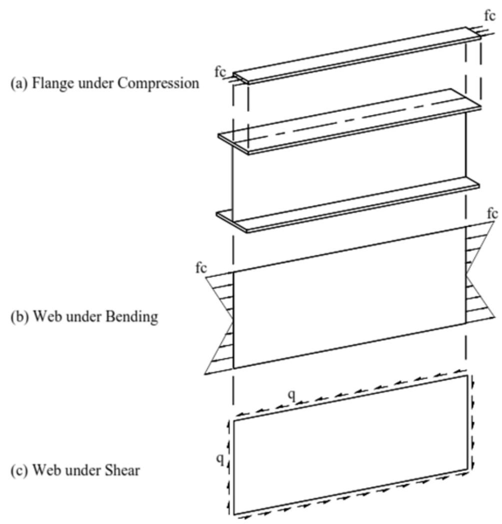
PLATE GIRDER BUCKLING



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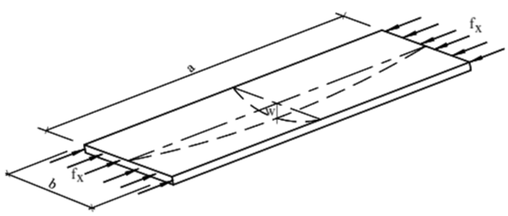
BUCKLING RESISTANCE

Consider a uniformly compressed plate of thickness t, width b, and length a simply supported along its four edges. Up to a certain load, the plate remains compressed in its own plane. However, as the load increases and reaches a critical value, the plane state of the plate becomes unstable. Further increase in load causes the plate to deflect laterally, resulting in the out-of-plane configuration. This phenomenon is referred to as plate buckling, and the stress that causes it is called the critical buckling stress.

BUCKLING RESISTANCE

The value of this buckling stress can be determined by applying structural mechanics theories to study the behavior of the plate. The assumptions used to solve this stability problem are those used in thin plate theory (Kirchhoff's theory):



BUCKLING RESISTANCE

1. Material is linear elastic, homogeneous and isotropic.
2. Plate is perfectly plane and initially stress free.
3. Thickness “t” of the plate is small compared to its other dimensions.
4. In-plane actions pass through its middle plane.
5. Transverse displacements w are small compared to the plate thickness.
6. Slopes of the deflected middle surfaces are small compared to unity.
7. Deformations are such that straight lines, initially normal to the middle plane, remain straight lines and normal to the deflected middle surface.
8. Stresses normal to the thickness of the plate are of a negligible order of magnitude.

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BUCKLING RESISTANCE

Based on these assumptions, the governing differential equation of the plate buckling is expressed as:

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + t f_x \frac{\partial^2 w}{\partial x^2} = 0 \ \ldots$$

where

f_x = normal stress

D = plate bending rigidity = E t^3 / 12(1 $-\nu^2$)

E = Elastic Modulus = 2100 t/cm^2

t = Plate thickness

ν = Poisson’s ratio = 0.3

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BUCKLING RESISTANCE

The solution of this equation gives the elastic buckling stress F_{cr} of the plate as:

$$F_{cr} = k_c \frac{\pi^2 E}{12(1-\upsilon^2)} \left(\frac{t}{b}\right)^2 = 1898\ k_c\ (t/b)^2..$$

where k_c = plate buckling factor which depends on the type of stress distribution, the edge support conditions, and the plate aspect ratio $\alpha = a/b$.
For the case considered it can be expressed as:

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BUCKLING RESISTANCE

$$k_c = (m / \alpha + \alpha / m)^2$$

where m = number of buckling half-waves in the longitudinal direction. For each value of m, there exists a corresponding buckling stress and a buckled configuration.
k_c is dependent on the ratio α for various values of m.
The buckling mode for values of $\alpha < \sqrt{2}$, has one half wave, for values $\sqrt{2} < \alpha < \sqrt{6}$, two half waves, etc.

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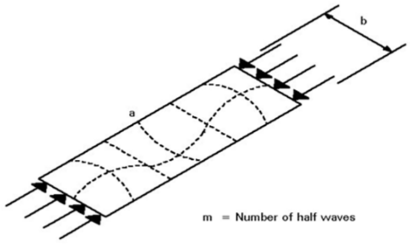
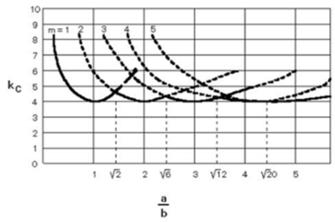
BUCKLING RESISTANCE





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BUCKLING RESISTANCE

Referring to the curve for m = 1, it is seen that k_c is large for small values of α and decreases as α increases until α = 1 (i.e., square plate) when k reaches its minimum value of 4. The value of k_c increases again as α increases. Similar behavior is obtained for other values of m. Therefore, k_c = 4 may be considered as valid for all values of m and is used as the basis for design.

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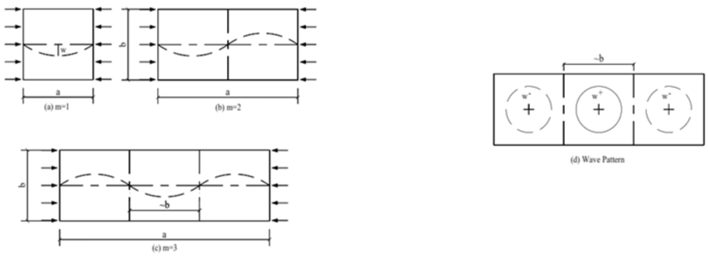
BUCKLING RESISTANCE

In a physical sense, Eqn. 5.16 can be interpreted to mean that a plate, simply supported on all four edges and uniformly compressed along the shorter sides, buckles in half – waves whose lengths approach the width of the plate.



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BUCKLING RESISTANCE

The above discussion applies to plates simply supported along their four edges. Plate girder sections may comprise plates, which are free along one longitudinal edge, and supported along the other edges, e.g., flange plates. Solution of the governing differential equation under these boundary conditions yields the value of the plate buckling factor kc = 0.425.

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BUCKLING RESISTANCE

Substituting the values of kc into Eqn. 5.5, the critical buckling stress is obtained as:

1- For a plate with simple supports, (kc =4)

$$F_{cr} = 7592 / (b/t)^2$$

2- For a plate with a free edge, (kc =0.425)

$$F_{cr} = 807 / (b/t)^2$$

The relationship between F_{cr} and (b/t) according to these equations is shown in the next figure.

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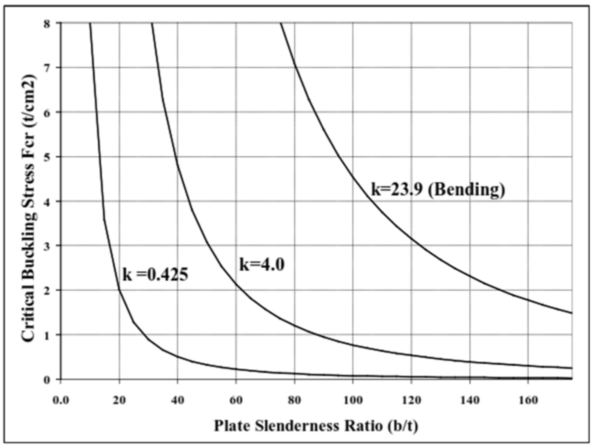
BUCKLING RESISTANCE



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BUCKLING RESISTANCE

Analogy with buckling of Axially Loaded Columns:
Compare and understand the differences between this buckling behavior of simply supported plates and that of axially loaded simply supported columns for which the critical load is given by:

$$P_{cr} = \pi^2 E I / L^2 .$$

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BUCKLING RESISTANCE

The simply supported column buckles into one half-wave of length L and the value of the critical buckling load is inversely proportional to L^2 and is independent of the column width. By contrast, because of the supports along the unloaded edges, a plate buckles into multiple half-waves the length of which approach the plate width b. The critical stress in the simply supported plate is inversely proportional to b^2 and independent of its length a.

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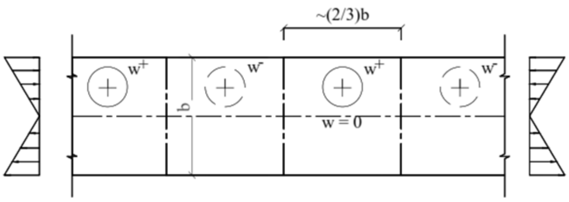
BUCKLING RESISTANCE

The above results can be extended to cover the general case of a plate subjected to a linearly varying compressive stress, e.g., due to bending moment in the plane of the plate.



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BUCKLING RESISTANCE

The critical elastic buckling stress for this case is expressed as:

$$F_{cr} = k_\sigma \frac{\pi^2 E}{12(1-\upsilon^2)} \left(\frac{t}{b}\right)^2 .$$

where the value of the plate buckling factor k_σ is given by:

$$k_\sigma = \frac{16}{\sqrt{(1+\psi)^2 + 0.112\,(1-\psi)^2} + (1+\psi)} \qquad \text{(for } 1 > \psi > -1\text{)}.$$

And $$k_\sigma = 5.98\,(1-\psi)^2 \qquad \text{(for } -1 > \psi > -2\text{)}.$$

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BUCKLING RESISTANCE

where $\psi = \sigma_2/\sigma_1$ = ratio of smaller stress σ_2 to larger stress σ_1. The value $\psi = 1$ corresponds to uniformly distributed compressive stress, where as the value $\psi = -1$ corresponds to the case of pure bending ($\sigma_2 = -\sigma_1$). The intermediate values, $-1 < \psi < +1$, correspond to combined bending and compression.

For the special case of pure bending, i.e., $\psi = -1$, the value of k_σ is equal to 25.9, giving:

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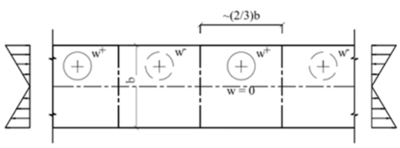
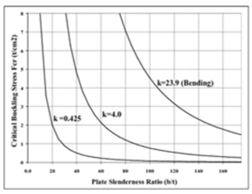
BUCKLING RESISTANCE

$$F_{cr} = 23.9\,\frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 = 45362\left(\frac{t}{b}\right)^2$$

A plot of the relationship between Fcr and (b/t) according to Eqn. 5.20 is shown in the previous Fig.





A typical buckling pattern of a plate initiated by bending of the plate.

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BUCKLING RESISTANCE

This bend-buckling is somewhat different than the buckling of a uniformly edge-compressed plate in that the out-of-plane deformation in the tensile zone of the plate is zero (shown by w = 0). The plate buckles in a single half-wave transversally (i.e., depth wise) and in multiple half-waves longitudinally (i.e., length wise). The lengths of the buckling waves approach 2/3 b.

BUCKLING RESISTANCE

Buckling under pure shear:
For a plate under the action of edge shear stresses, the stresses are equivalent to tension and compression stresses that are equal in magnitude to the shear stresses but inclined at 45^o. The compressive stresses may cause the plate to buckle. The buckling mode is composed of multiple wave forms which are skewed with respect to the edges. The half-wave length is equal to about 1.25 b for long simply supported plates.

BUCKLING RESISTANCE

According to the elastic buckling theory, the critical buckling shear stress can be expressed as:

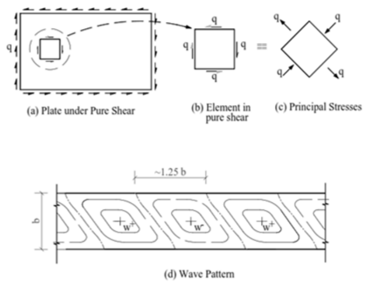
$$q_{cr} = k_q \frac{\pi^2 E}{12(1-\upsilon^2)} \left(\frac{t}{b}\right)^2 = 1898\, k_q \left(\frac{t}{b}\right)^2$$

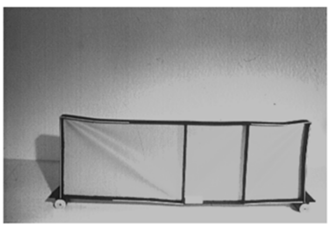
where k_q is a shear buckling factor calculated from elastic buckling theory according to the plate aspect ratio α = a/b as follows:

$$k_q = 4.00 + 5.34 / \alpha^2 \qquad \alpha < 1\ldots\ldots$$

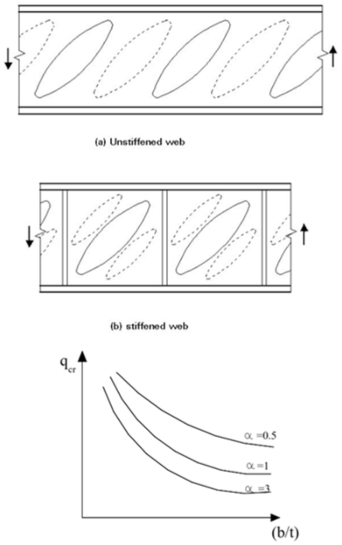
$$k_q = 5.34 + 4.00 / \alpha^2 \qquad \alpha > 1\ldots\ldots$$

BUCKLING RESISTANCE





BUCKLING RESISTANCE



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ACTUAL RESISTANCE

The buckling theory described in the previous section is based on the earlier assumptions that are never fulfilled in real structures. The consequences for the buckling behavior when each of these assumptions is not valid are now discussed.

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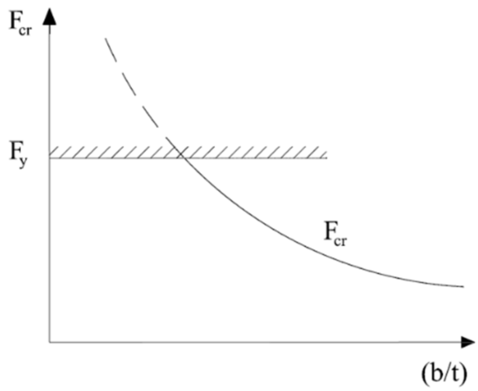
Effect of Inelastic Behavior

The first assumption of linear elastic behavior of the material is obviously not valid when the value of F_{cr} according to these equations exceeds the material yield strength F_y. This behavior is typical for thick plate panels having low (b/t) ratios. In this case failure is governed by yielding rather than buckling. If the material is considered to behave as linear elastic-ideal plastic, the buckling curve must be cut off at the level of the yield stress F_y.

Effect of Inelastic Behavior



Effect of Inelastic Behavior

Accordingly, two regions must be considered for establishing strength:

1. for large (b/t) values: $F_{cr} < F_y$, i.e., Elastic buckling governs the design.
2. for low (b/t) values: $F_{cr} \geq F_y$, i.e, Yielding governs the design.

For design purposes, it is convenient to define a plate slenderness parameter λ in terms of the ratio of the yield stress to the critical stress F_y/F_{cr} as:

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Effect of Inelastic Behavior

$$\lambda = \sqrt{\frac{F_y}{F_{cr}}} = \sqrt{\frac{F_y}{k\,(1898)\left(\frac{t}{b}\right)^2}} = \left(\frac{b}{t}\right)\sqrt{\frac{F_y}{1898\,k}} \quad ....$$

A plot of F_{cr}/F_y versus λ is shown. Curve (a) represents the theoretical buckling curve defined by the Eqn., while the horizontal line at $F_{cr} = F_y$ represents the yield condition. The value $\lambda = 1$ represents the limit between elastic buckling and yield. Consequently, the plate fails due to elastic buckling when $\lambda > 1$ and due to yield when $\lambda < 1$.

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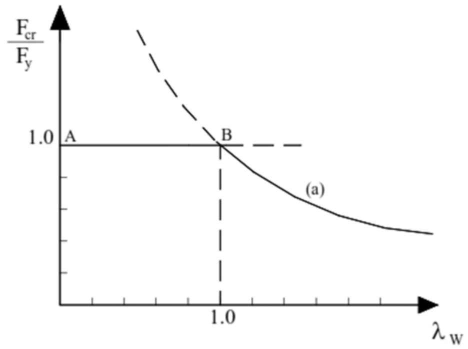
Effect of Inelastic Behavior



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Effect of Imperfections and Residual Stresses

The second and fourth assumptions of a plate without geometrical imperfections and residual stresses are also never fulfilled in real structures. Plates in fabricated structures are likely to have some initial out-of-plane deviations. When the plate is loaded, these deviations will start growing in depth and thus cause additional stresses on the cross section.

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Effect of Imperfections and Residual Stresses

Furthermore, steel plates as well as rolled sections contain residual stresses. Residual stresses in rolled sections are mainly caused by uneven cooling after hot rolling. Plates in welded plate girders are subjected to high temperatures during flame-cutting and welding. Shrinkage due to cooling of the hot areas is resisted by the remaining cold parts of the cross section.

Effect of Imperfections and Residual Stresses

As a result, the areas adjacent to the weld or flame cut are subjected to high tensile strains which may be several times the yield strain, and the rest of the cross-section is subjected to compression. As compressive and tensile residual stresses in the cross-section balance, residual stresses do not cause any resultant axial force or bending moment on the cross-section.

Effect of Imperfections and Residual Stresses

However, those parts of the cross section where the residual stress is of the same nature as the applied stress will reach yield earlier. With further loading these yielded parts will not contribute any resistance to the cross section and thus the effective stiffness, and consequently the plate buckling strength, will be reduced.

Effect of Imperfections and Residual Stresses

Residual stresses are less important for plates subjected to shear or bending stress than plates under compression because the applied stresses and the residual stresses are likely to be of a different nature in different parts of the plates.

Effect of Imperfections and Residual Stresses

Tests have shown that the reduction in plate buckling strength due to imperfections and residual stresses is most pronounced for plates with intermediate values of (b/t). For design purposes, this effect is considered by using a reduced value of the limit plate slenderness λ0 < 1.

Effect of Imperfections and Residual Stresses

Because of statistical variations in material properties and imperfections which are not sufficiently well known to be quantified accurately, the appropriate value of λ_o differs substantially from country to country. A review of the international design codes shows that λ_o varies approximately from 0.6 to 0.9. ECP has adopted the following limiting values for the plate slenderness parameter:

Effect of Imperfections and Residual Stresses

1. λ_o = 0.74 for Class 2 elements in compression.
2. λ_o = 0.90 for Class 2 elements in bending.
3. λ_o = 0.80 for elements under pure shear.

These values can be used to calculate the limiting slenderness ratios of different parts in a plate girder section as follows:

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Quick Review

- Introduction.
- Plate Girder Components.
- Design Considerations:
  - Bucking due to shear
  - Buckling due to bending
  - Stress calculations.
- Inelastic behavior.
- Effect of Imperfections.

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