Theorem: For task i, the minimum nonzero separation of a task instance release time from the corresponding frame start time is $\gcd(f, T_i)$.

Proof:

Let $g = \gcd(f, T_i)$, it follows from the definition of the greatest common divisor (gcd) that g must divide each of f and T_i. Let i be a task with zero phasing. Now, assume that this theorem is violated for certain integers m and n, such that the nth release of task i occurs in the mth frame and the difference of the start of the mth frame and the release of the nth instance is a nonzero value less than g, that is

$$0 < (m \times f - n \times T_i) < g$$

Dividing this expression throughout by g, we get

$$0 < (m \times f/g - n \times T_i/g) < 1$$

However, both f/g and T_i/g are integers because $g = \gcd(f, T_i)$. Therefore we can write $f/g = I_1$, and $T_i/g = I_2$ for some integer values I_1 and I_2. Substituting in the above inequality we get

$$0 < (m \times I_1 - n \times I_2) < 1$$

Since $m \times I_1$ and $n \times I_2$ are both integers, their difference cannot be a fractional value between 0 and 1. Therefore this inequality can never be satisfied.

It can therefore be concluded that the minimum time between a frame boundary and the arrival of the corresponding instance of task cannot be less than $\gcd(f, T_i)$. [Actually it cannot be less than any common divisor, thus its minimum value is the gcd.]

□

(Proof adapted from notes by Rajib Mall).