METALLIC STRUCTURES

COMPRESSION MEMBERS

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TOPICS

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INTRODUCTION

- Compression Members are those subjected to PURE COMPRESSION forces.
- Design procedure is similar to the design of Tension Members except for STABILITY (BUCKLING) phenomena.
- There is a Stability Problem. The acting compression tends to bend the member off its straight alignment.

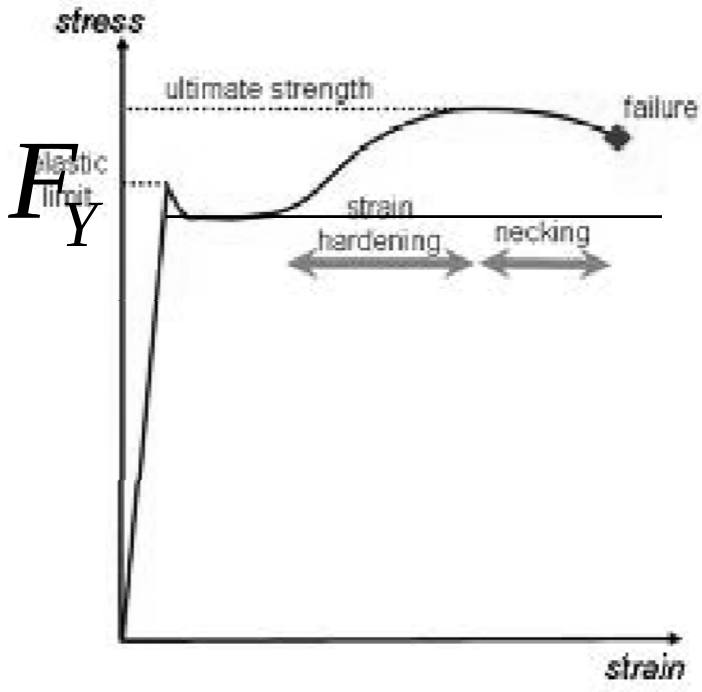
$$\Longrightarrow \quad f_{act} \leq f_{all}$$

- Because of the above the stiffness limit is very strict.
- Applications $\Longrightarrow$ Truss members, Pin ended columns, knee bracing, …

BEHAVIOR OF COMPRESSION MEMBERS

- For CONCENTRIC compression forces, the resulting stress is a uniform stress equally distributed over the member area.

$$f_{act} = \frac{C}{A}$$



BEHAVIOR OF COMPRESSION MEMBERS

- For bolted construction, the bolt shank is assumed to full the hole. Therefore, the gross area is always used in assessing the actual stress.

$$f_{c,act} = \frac{C}{A_g}$$

BEHAVIOR OF COMPRESSION MEMBERS

The well known moment-curvature relation

$$\frac{M}{EI} = -\frac{d^2 V}{d Z^2} \text{; Substituting } M = P.V, \text{ Diff. equ. would be :}$$

$$\frac{d^2 V}{d Z^2} + K^2 V = 0 \text{; where } K^2 = P/EI$$

The solution of the Equation :

$$V(Z) = A \sin KZ + B \cos KZ$$

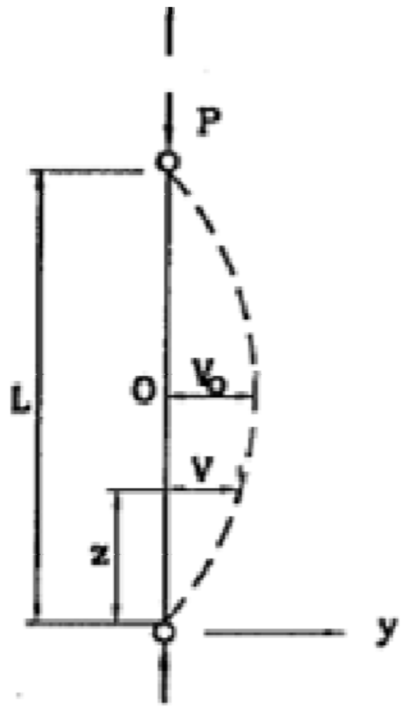
wher A and B are constants depends on boundary conditions

For $V(0) = 0$ and $V(L) = 0$ Therefore $B = 0$ and $A \sin KL = 0$

For non-trivial solution $(A \neq 0)$ then :

$$K = \frac{n\,\pi}{L}, n = 1, 2, 3, ...; \text{or } P = \frac{n^2\,\pi^2}{L^2}.EI$$

Smallest Load, $n = 1$ thus $P_{crit.} = P_{Euler} = \dfrac{\pi^2 EI}{L^2}$



BEHAVIOR OF COMPRESSION MEMBERS

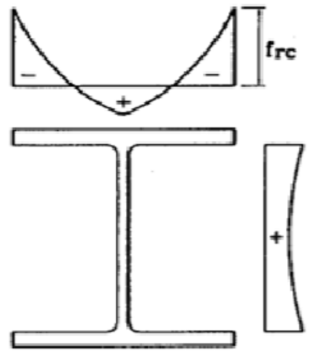
- As the load increases, the stress increases. But failure generally occurs at a stress much lower that the yield stress . Failure is SUDDEN.

$$P_{critical} = \frac{\pi^2 \; E \; I}{(K \; L)^2}$$

- K = buckling length factor
- L = member length
- E = Modulus of Elasticity
- I = Section moment Inertia

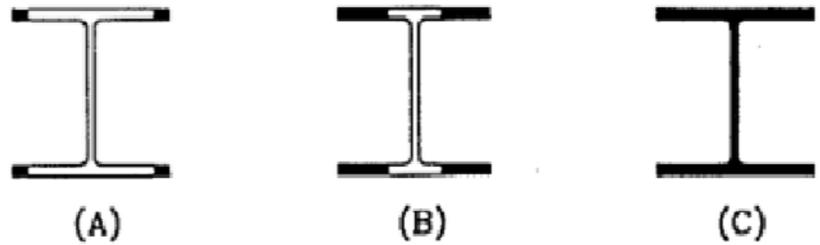
BEHAVIOR OF COMPRESSION MEMBERS

Residual Stresses

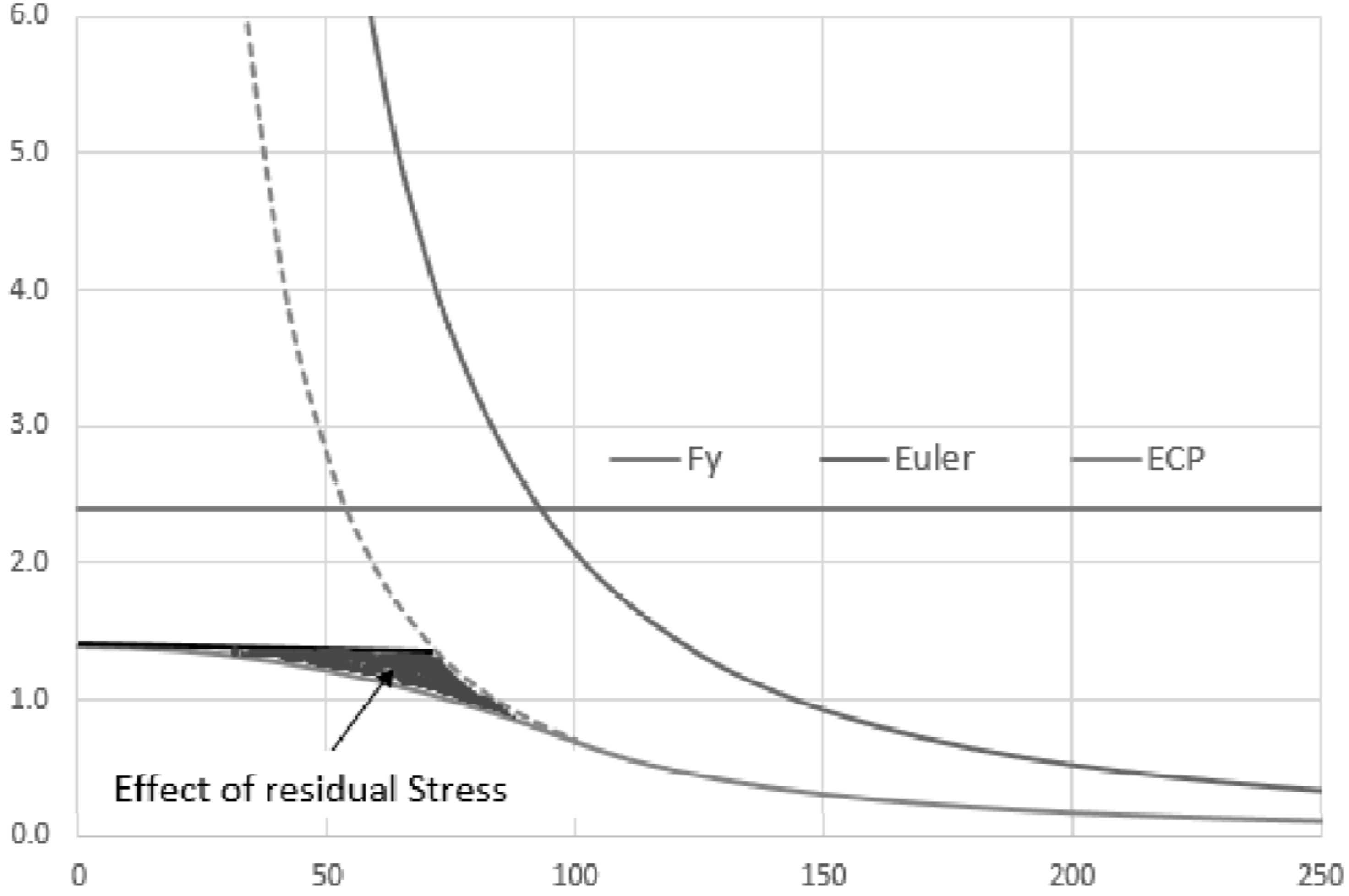


- Due to the non-uniform cooling of section

- Due to that plastification of the section will start at the tips of flanges at stress below the yield stress.



BEHAVIOR OF COMPRESSION MEMBERS



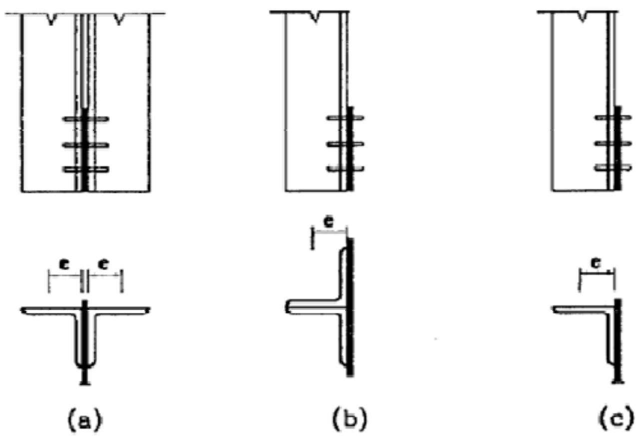
ALLOWABLE STRESSES

- Case I Loading (Main Loads)

$$F_c = \frac{7500}{(\frac{KL}{i})^2} \quad \Rightarrow \quad \frac{KL}{i} \leq 100$$

$$F_c = 0.58F_Y - \frac{(0.58F_Y - 0.75)}{10,000} \times (\frac{KL}{i})^2 \quad \Rightarrow \quad \frac{KL}{i} \geq 100$$

$$(St.37 \rightarrow \quad F_c = 1.4 - 0.000065 \times (\frac{KL}{i})^2$$



- Case II Loading (Secondary Loads)

Increase allowable stresses by 20%

- For eccentrically loaded members:

Reduce the allowable stress by 40%

(a) No reduction; (b) and (c) reduce 40%

ALLOWABLE STRESSES

- For λ = slenderness ratio = $k\ell/r < 100$ (see Chapter 4 for definition of terms):

$$F_c = 0.58F_y - \frac{(0.58F_y - 0.75)}{10^4}\lambda^2 \qquad \text{2.11}$$

Grade of Steel	F_c (t/cm^2)		
	$t \leq 40$ mm	40 mm $< t \leq 100$ mm	
St 37	$F_c = (1.4 - 0.000065\lambda^2)$	$F_c = (1.3 - 0.000055\lambda^2)$	**2.12**
St 44	$F_c = (1.6 - 0.000085\lambda^2)$	$F_c = (1.5 - 0.000075\lambda^2)$	**2.13**
St 52	$F_c = (2.1 - 0.000135\lambda^2)$	$F_c = (2.0 - 0.000125\lambda^2)$	**2.14**

For all grades of steel:

For $\lambda = k\ell/r \geq 100$:

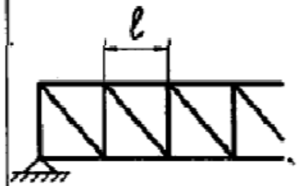
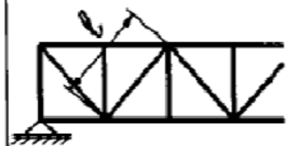
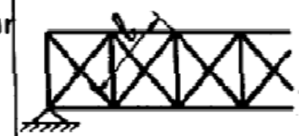
$$F_c = 7500/\lambda^2 \qquad \text{2.15}$$

BEHAVIOR OF COMPRESSION MEMBERS

- Buckling length of any member needs to be evaluated in BOTH planes (in-plane and out-of plane)
- Buckling length about any Axis is the buckling length in the plane PERPENDICULAR to that axis.
- All truss joints are assumed as hinged (partial rigidity due to connection is neglected).
- In general, in-plane buckling length of a truss member = geometric length of the member.

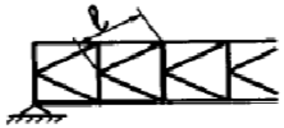
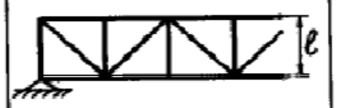
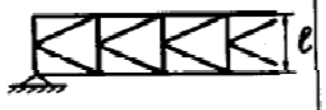
BEHAVIOR OF COMPRESSION MEMBERS

Table (4.4) Buckling Length of Compression Members in Buildings and in Bridge Bracing Systems

Member		In-Plane	Out-of-Plane	
			Compression Chord Effectively Braced	Compression Chord Unbraced
Chords		ℓ	ℓ	0.75 span (Clause 4.3.2.2)
Diagonals –Single Triangulated web system		ℓ	ℓ	$1.2\,\ell$
–Multiple Intersected web rectangular system adequately connected		$0.5\,\ell$	$0.75\,\ell$	ℓ

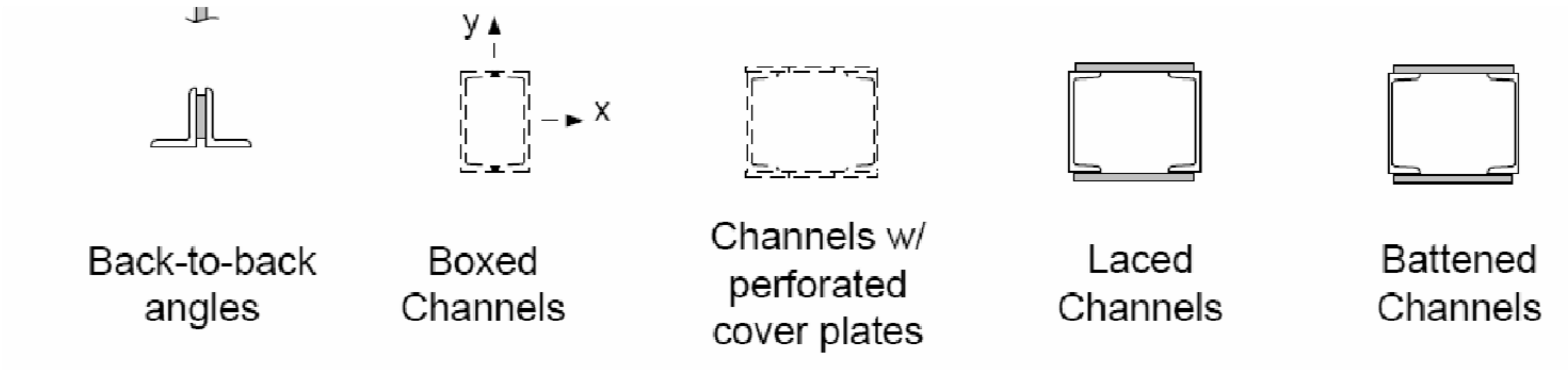
BEHAVIOR OF COMPRESSION MEMBERS

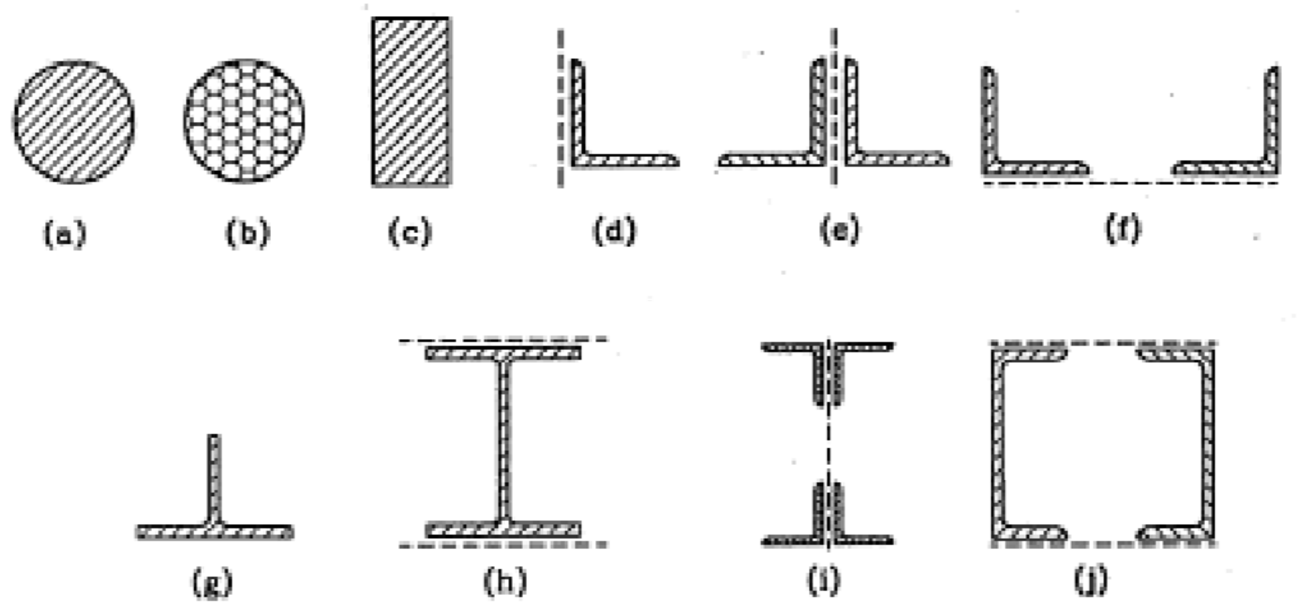
Table (4.4) Buckling Length of Compression Members in Buildings and in Bridge Bracing Systems (Cont.)

Member		In-Plane	Out-of-Plane: Compression Chord Effectively Braced	Out-of-Plane: Compression Chord Unbraced
Diagonals -Multiple intersected web trapezoidal system adequately connected		ℓ	$0.8\,\ell_d$	———
- K-system		ℓ	$1.2\,\ell$	$1.5\,\ell$
Vertical members -Single triangulated web system		ℓ	ℓ	$1.2\,\ell$
-K-intersected web system		$0.5\,\ell$	$(0.75+0,25\frac{N_s}{N_L})\ell$	$(0.90+0.30\frac{N_s}{N_L})\ell$

N_s = Smaller value of compression force
N_L = Larger value of compression force

CROSS SECTION TYPES





STIFFNESS LIMITATION

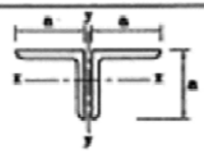
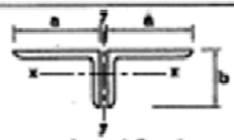
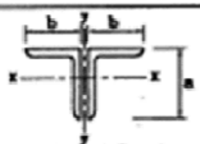
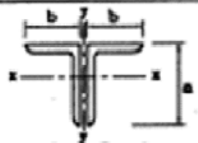
$$\left(\frac{KL}{i}\right)_{\max} \leq 180 \qquad i = \sqrt{\frac{I}{A}}$$

Compute slenderness ratio in-plane and out of plane

Table(4.1) Maximum Slenderness Ratio for Compression Members

Member	λ_{max}
Buildings:	
Compression members	180
Bracing systems and secondary members	200
Bridges:	
Compression members in railway bridges	90
Compression members in roadway bridges	110
Bracing systems	140

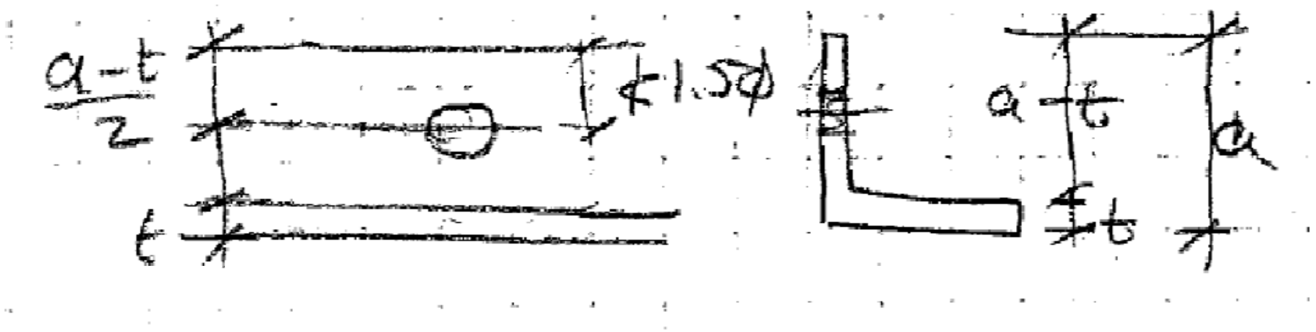
STIFFNESS LIMITATION

CASE	SECTION OF MEMBER	i_x or i_v	i_y or i_u
1		$i_x = 0.3\ a$	--
2	 a : b = 1.5 : 1	$i_x = 0.28\ b$	$i_y = 0.48\ a$
3	 a : b = 1.5 : 1	--	$i_y = 0.3\ a$
4	 a : b = 2 : 1	--	$i_y = 0.3\ a$
5		$i_v = 0.2\ a$	--
6	 a : b = 1.5 : 1	$i_v = 0.14\ a$	--
7	 a : b = 2 : 1	$i_v = 0.1\ a$	--
8		$i_v = 0.385\ a$	--

CONSTRUCTION CONDITION

- To allow for proper installation and tightening of bolts (use only in bolted connections).

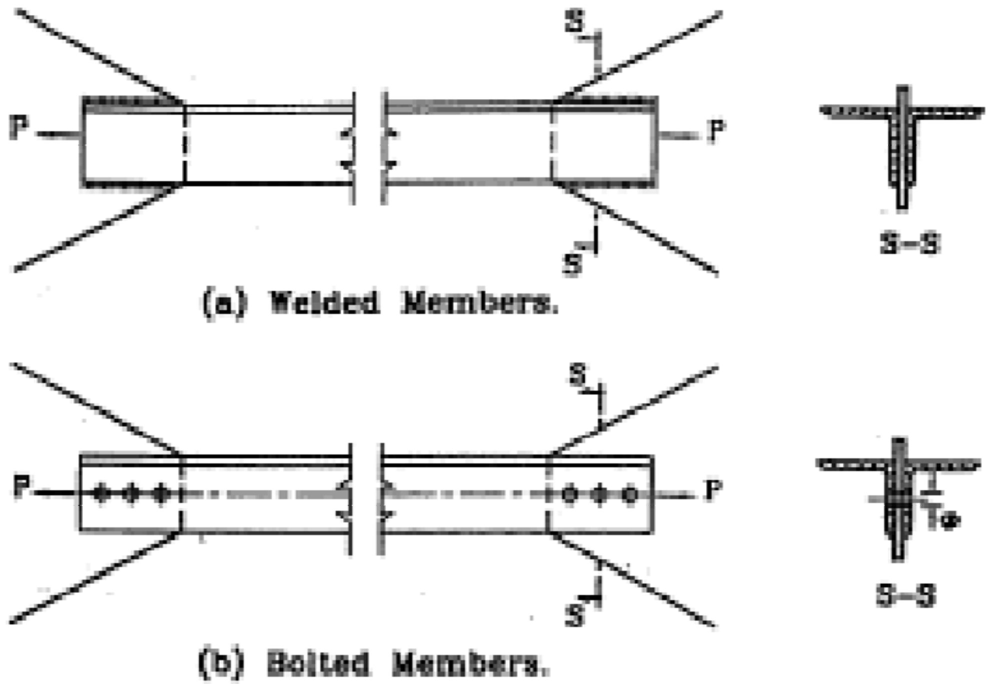
$$a - t \geq 3d_b$$



ACTUAL STRESSES

- Welded or Bolted Connection

$$f_{ca} = \frac{C}{A_g}$$



DESIGN STEPS

- Determine
  - DF (Compression Force), Load Case (I or II)
  - Member location, Length (L_g), Bolted or Welded
- Determine $L_{in\ plane}$ and $L_{out\ of\ plane}$
- Choose section type (1L, 2L back to back, 2L star shape).
- Stiffness condition (get minimum “a”)
- Construction condition (bolted), (get minimum “a-t”)
- Obtain an approximate area

$$A_{app} = \frac{DF}{F_{all.} \times 0.6 \times 1.2}$$

Design Force (tons)	< 5	5-10	10-25	> 25
Allowable Stress F_c (t/cm^2)	0.6	0.75	0.8	0.95

0.6 (non symmetric section), 1.2 (if case II)

DESIGN STEPS

- Choose a suitable section from tables
  - Use minimum “a”
  - Use A_{app}
- Check of Safety
  - Actual Stress
  - Allowable Stress

$$f_{ca} = \frac{C}{A_g} \qquad F_c = f\left(\frac{KL}{i}\right)$$

$$f_{ca} \leq F_c$$

Tension – Compression Members

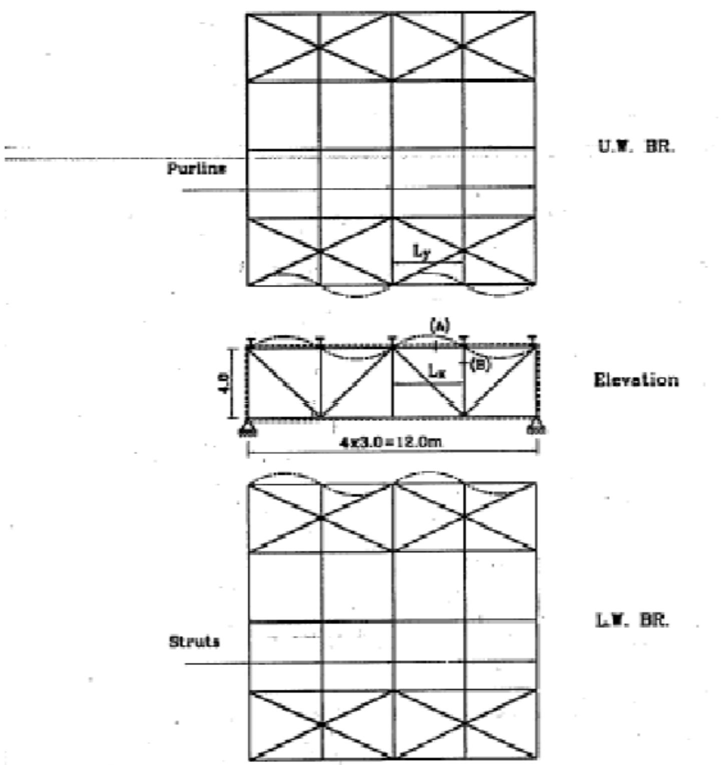
- Start design with either tension or compression force
- Check safety for the other straining action based on the chosen section.
- Use slenderness ratio of compression members in all cases.
- In general
    - If $T > 2C$ start with the tension member design and check safety on the compression force
    - If $T < 2\ C$ start with the compression member design and check safety on the tension force

EXAMPLES

Example (3.1):

Design a top compression member (A) if the design force D.F. = - 28 tons (Case of loading II) and its length, l = 300 cms (ϕ = 20 mms).

Solution



Type of Cross Section:

The member being a top chord member : choose a symm. section (2 <^{s} back to back).

Buckling Length:

From figure $l_x = l_y$ = 300 cms

choose 2 angles of equal legs back to back.

Stiffness Condition:

$l / i \leq 180$, $i_x = 0.30\ a$ (Table 2.2)

$$\therefore\ a \geq \frac{300}{0.3 \times 180} \geq 5.56 \text{ cms}$$

Construction Condition:

$(a-t) \geq 3\ \phi \geq 6.0$ cms.

Req. Approx. Area:

Assume $f_{av.} = 0.95\ t/cm^2$.

$$\text{Area}_{1<\text{ gross}} = \frac{28.0}{2 \times 0.95 \times 1.2} = 12.27\ cm^2$$

↑ 2 <^{s} ↑ av. stress ↖ (Case of loading II)

Check of Stresses and provisions for local buckling :-

. Try 2 <^{s} (80x80x8) $A_{1<} = 12.30\ cm^2$

b/t = 8.0/1.0 = 8.0 <14.8 (Table 3.2) i.e. No local buckling

$$l_x / i_x = \frac{300}{0.30 \times 8} = 125 < 180 \text{ (o.k)}$$

$$F_c = \frac{7500}{(125)^2} \times 1.2 = 0.576\ t/cm^2$$

$$f_c = \frac{28}{2 \times 12.30} = 1.138\ t/cm^2 > F_c \text{ (unsafe)}$$

. Try 2 <^{s} (100x100x10) $A_{1<} = 19.20\ cm^2$

$$l_x / i_x = \frac{300}{0.30 \times 10} = 100 \quad \therefore\ F_c = \frac{7500}{(100)^2} \times 1.2 = 0.9\ t/cm^2$$

$f_c = 28 / (2 \times 19.2) = 0.73\ t/cm^2 < F_c$ $\therefore$ Safe and economic