# Introduction to Computational Electromagnetics
# Lecture 3: 2D Electromagnetic Scattering
# Part 2: The Magnetic Field Integral Equation

ELC 657 – Spring 2014

Department of Electronics and Communications Engineering

Faculty of Engineering – Cairo University

# Outline

1. TM Polarization
   - Problem Formulation
   - MoM Procedure
   - Sample Results

2. TE Polarization
   - Problem Formulation
   - MoM Procedure
   - Sample Results

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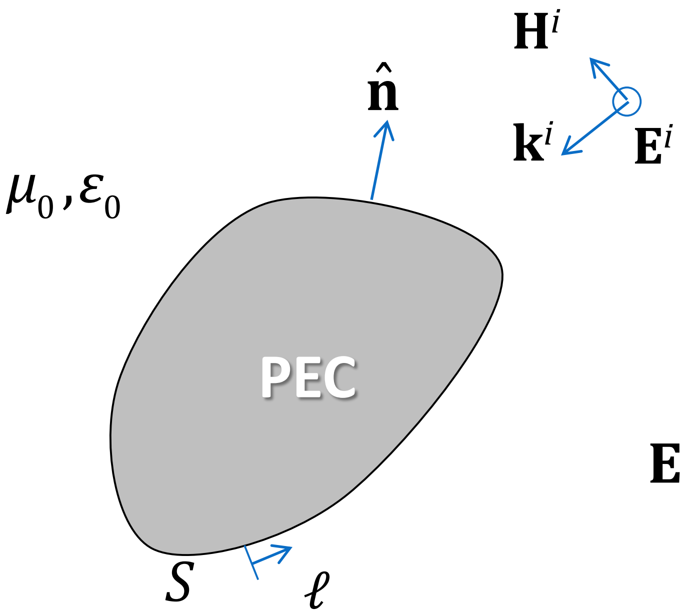
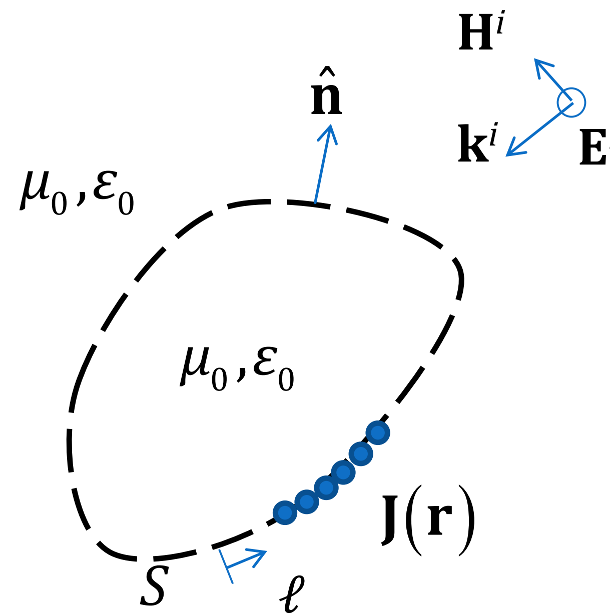
Problem Formulation

# Equivalent Problem




$$\mathbf{E}(\mathbf{r}), \mathbf{H}(\mathbf{r}) = ??$$




$$\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{E}(\mathbf{r}) = \mathbf{0}, \quad \mathbf{r} \in S$$

$$\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{H}(\mathbf{r}) = \mathbf{J}(\mathbf{r}), \quad \mathbf{r} \in S^+$$

$$\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{H}(\mathbf{r}) = \mathbf{0}, \qquad \mathbf{r} \in S^-$$

It is required to find the scattered electric and magnetic electric field everywhere due to a conducting cylinder illuminated by a plane wave.

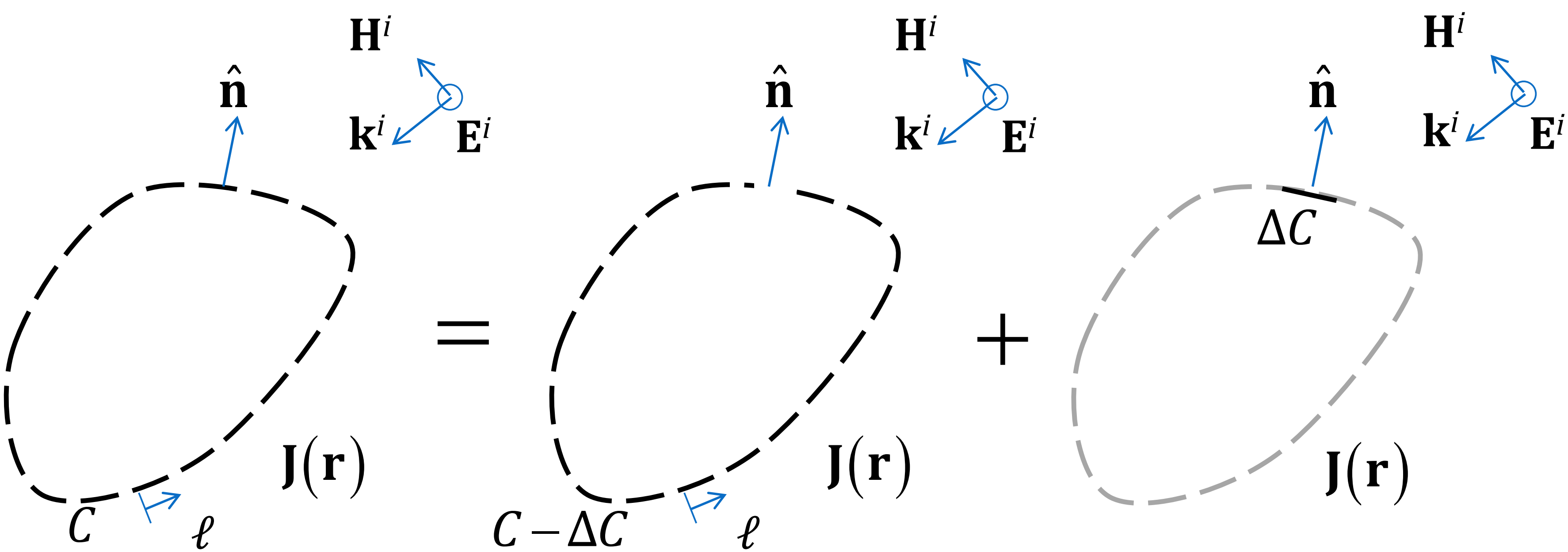
Problem Formulation

# The Magnetic Field and Electric Current

$$\hat{\mathbf{n}}\times\mathbf{H}^{+}=\hat{\mathbf{n}}\times\left[\mathbf{H}^{i+}+\mathbf{H}^{s+}\right]=\hat{\mathbf{n}}\times\left[\mathbf{H}^{i+}+\mathbf{H}_{ns}^{s+}+\mathbf{H}_{s}^{s+}\right]=\mathbf{J}$$

$$\hat{\mathbf{n}}\times\mathbf{H}^{-}=\hat{\mathbf{n}}\times\left[\mathbf{H}^{i-}+\mathbf{H}^{s-}\right]=\hat{\mathbf{n}}\times\left[\mathbf{H}^{i-}+\mathbf{H}_{ns}^{s-}+\mathbf{H}_{s}^{s-}\right]=\mathbf{0}$$

$$\hat{\mathbf{n}}\times\left[\mathbf{H}_{s}^{s+}-\mathbf{H}_{s}^{s-}\right]=\mathbf{J} \quad \Rightarrow \quad \boxed{\hat{\mathbf{n}}\times\mathbf{H}_{s}^{s\pm}=\pm\tfrac{1}{2}\mathbf{J}}$$



Problem Formulation

# Maue's Magnetic Field Integral Equation

$$\boxed{\hat{\mathbf{n}} \times \left[ \mathbf{H}^{i}(\mathbf{r}) + \mathbf{H}^{s}_{ns}(\mathbf{r};\mathbf{J}) \right] = \tfrac{1}{2}\mathbf{J}(\mathbf{r}),\ \mathbf{r} \in S}$$

$$-\tfrac{1}{2}\mathbf{J}(\mathbf{r}) + \frac{1}{\mu_0}\hat{\mathbf{n}} \times \nabla \times \int_{S-\Delta S} \mathbf{J}(\mathbf{r}')\, g(\mathbf{r},\mathbf{r}')\, ds' = -\hat{\mathbf{n}} \times \mathbf{H}^{i}(\mathbf{r}),\ \mathbf{r} \in S$$

$$\nabla \times (\mathbf{J}g) = \cancel{g\nabla \times \mathbf{J}} - \mathbf{J} \times \nabla g$$

$$\nabla \times (\mathbf{J}g) = -\mathbf{J} \times \nabla g = \mathbf{J} \times \nabla' g$$

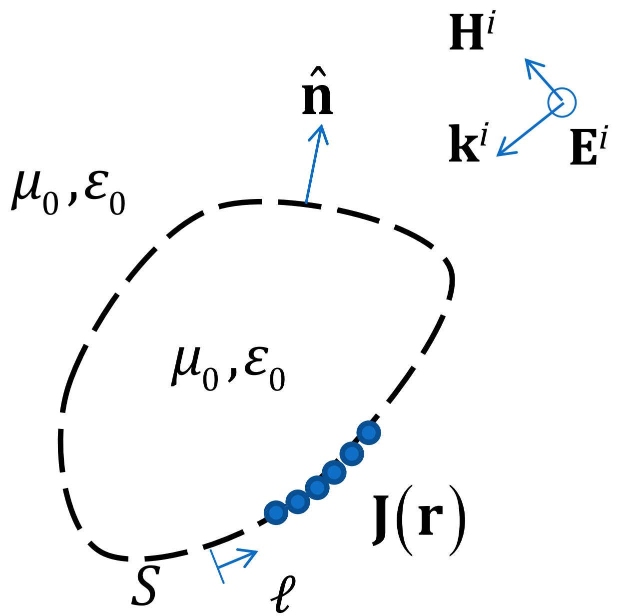
$$-\tfrac{1}{2}\mathbf{J}(\mathbf{r}) + \frac{1}{\mu_0}\hat{\mathbf{n}} \times \int_{S-\Delta S} \mathbf{J}(\mathbf{r}') \times \nabla' g(\mathbf{r},\mathbf{r}')\, ds' = -\hat{\mathbf{n}} \times \mathbf{H}^{i}(\mathbf{r}),\ \mathbf{r} \in S$$

Problem Formulation

# Operator Equation

$$\mathbf{J}(\boldsymbol{\rho}) = J_z(\boldsymbol{\rho})\hat{\mathbf{z}}$$

$$\boxed{-\tfrac{1}{2}J_z(\boldsymbol{\rho}) + \frac{k_0}{4j}\int_{C-\Delta C} J_z(\boldsymbol{\rho}')\hat{\mathbf{n}}\cdot\hat{\mathbf{R}}H_1^{(2)}\left(k_0\left|\boldsymbol{\rho}-\boldsymbol{\rho}'\right|\right)dl' = -H_1^i(\boldsymbol{\rho}), \qquad \boldsymbol{\rho}\in C}$$




$$L\left\{J_z(\boldsymbol{\rho})\right\} = -H_1^i(\boldsymbol{\rho}),\ \boldsymbol{\rho}\in C$$

$$H_1^i = \hat{\mathbf{l}}\cdot\mathbf{H}^i = \frac{1}{\eta_0}\hat{\mathbf{l}}\cdot\hat{\mathbf{k}}^i\times\mathbf{E}^i,\ \boldsymbol{\rho}\in C$$

$$E_z^i(\boldsymbol{\rho}) = E_0 e^{-j\mathbf{k}^i\cdot\boldsymbol{\rho}} = E_0 e^{+jk_0\left(\cos\varphi^i x + \sin\varphi^i y\right)}$$

Based on the previous derivations, the MFIE formulation can be used only for closed objects.

The MoM Procedure

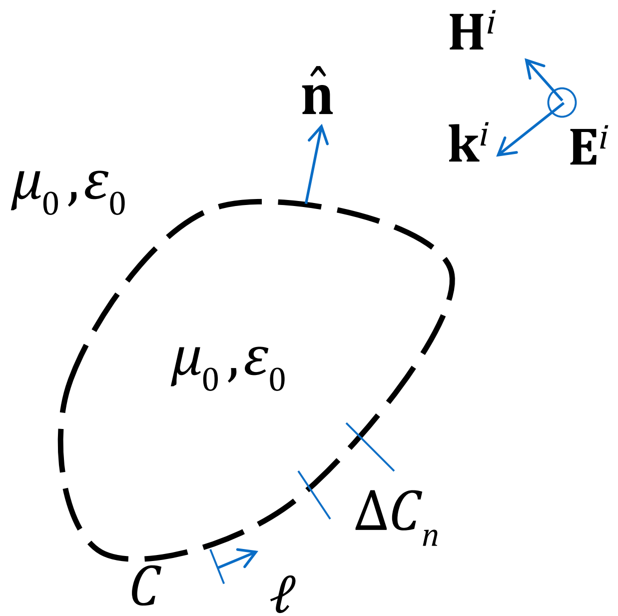
# Expansion, Testing and MoM Matrix Equation

$$\Pi_n(\boldsymbol{\rho}) = \begin{cases} 1, & \boldsymbol{\rho} \in \Delta C_n \\ 0, & \text{otherwise} \end{cases} \qquad t_m(\boldsymbol{\rho}) = \delta(\boldsymbol{\rho} - \boldsymbol{\rho}_m)$$

$$\sum_{n=1}^{N} I_n Z_{mn} = V_m,\ m = 1,2,\ldots,N$$

$$Z_{mn} = -\tfrac{1}{2}\delta_{mn} + \frac{k_0}{4j}\int_{\Delta C_n} \hat{\mathbf{n}}_m \cdot \hat{\mathbf{R}} H_1^{(2)}\left(k_0 \left|\boldsymbol{\rho}_m - \boldsymbol{\rho}'\right|\right) dl'$$

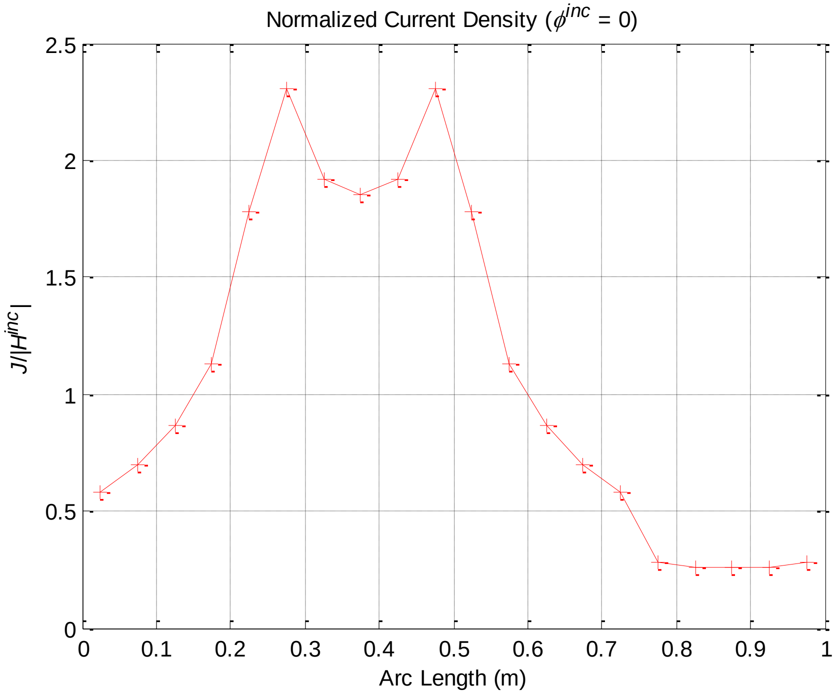
$$V_m = -\frac{1}{\eta_0} E_0 e^{+jk_0\left(\cos\varphi^i x_m + \sin\varphi^i y_m\right)} \left(\cos\varphi^i \hat{\mathbf{y}} - \sin\varphi^i \hat{\mathbf{x}}\right) \cdot \hat{\mathbf{l}}_m$$

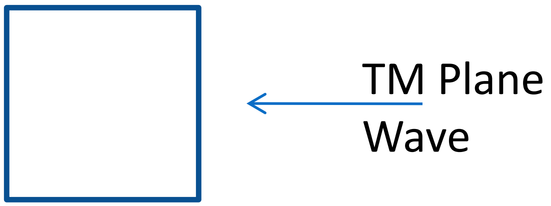



Notice that there are no singularity issues in this case. However, near-self terms may still need singularity extraction.

Sample Results

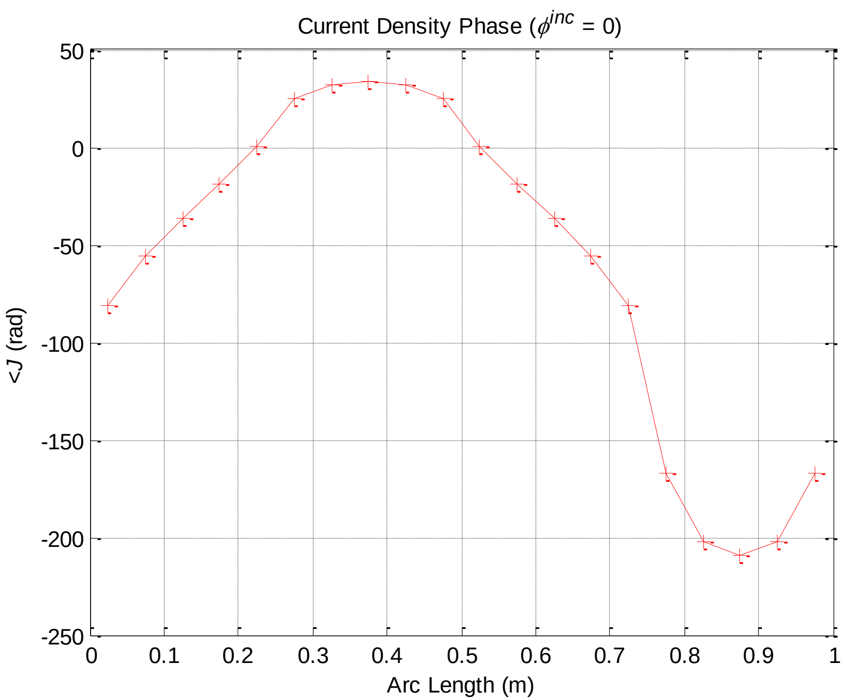
# Sample Case 1: Quarter-Wavelength PEC Square




$f$ = 300 MHz

$\lambda$ = 1 m



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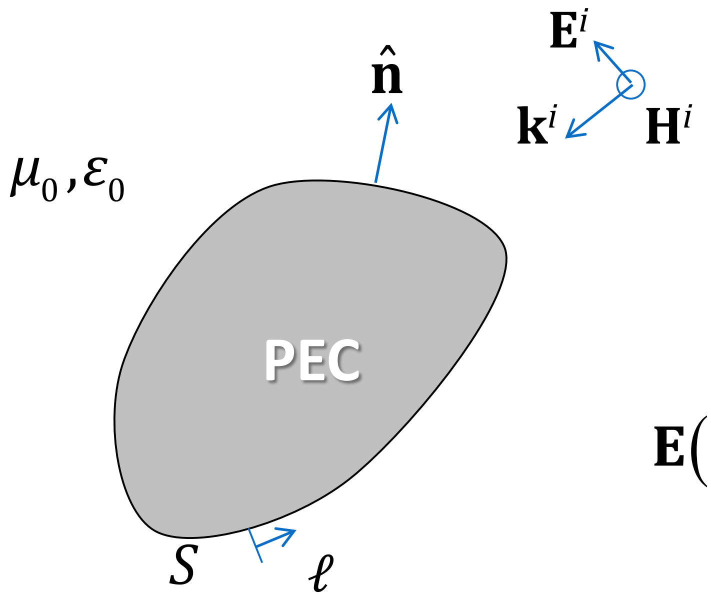
Problem Formulation

# Equivalent Problem

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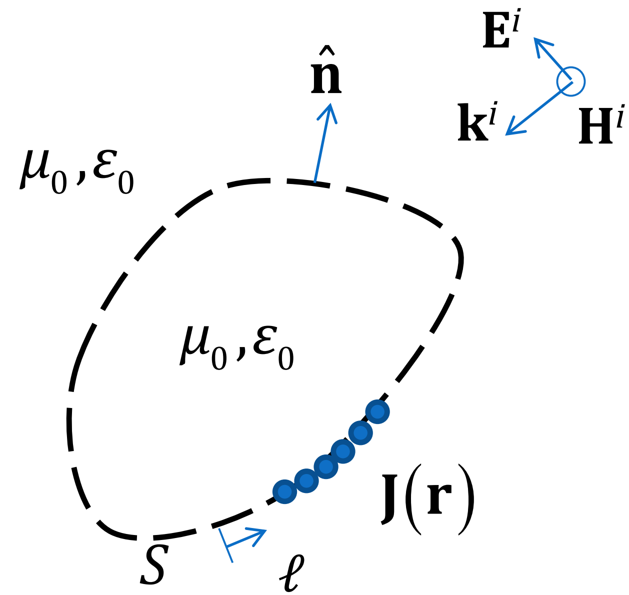


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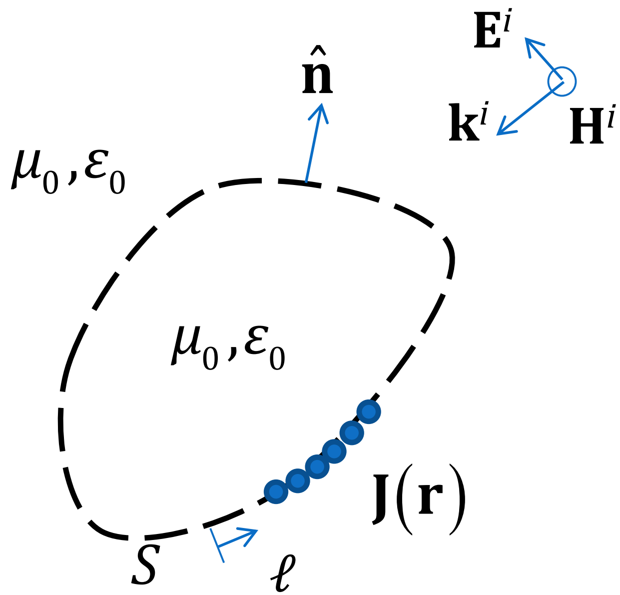
It is required to find the scattered electric and magnetic electric field everywhere due to a conducting cylinder illuminated by a plane wave.

Problem Formulation

# Operator Equation

$$\mathbf{J}(\boldsymbol{\rho}) = J_l(\boldsymbol{\rho})\hat{\mathbf{l}}$$

$$\boxed{\begin{aligned} -\tfrac{1}{2}J_l(\boldsymbol{\rho}) + \frac{k_0}{4j}\int_{C-\Delta C} J_l(\boldsymbol{\rho})\hat{\mathbf{n}}'\cdot\hat{\mathbf{R}}H_1^{(2)}\left(k_0\left|\boldsymbol{\rho}-\boldsymbol{\rho}'\right|\right)dl' \\ = H_z^i(\boldsymbol{\rho}), \qquad \boldsymbol{\rho}\in C \end{aligned}}$$

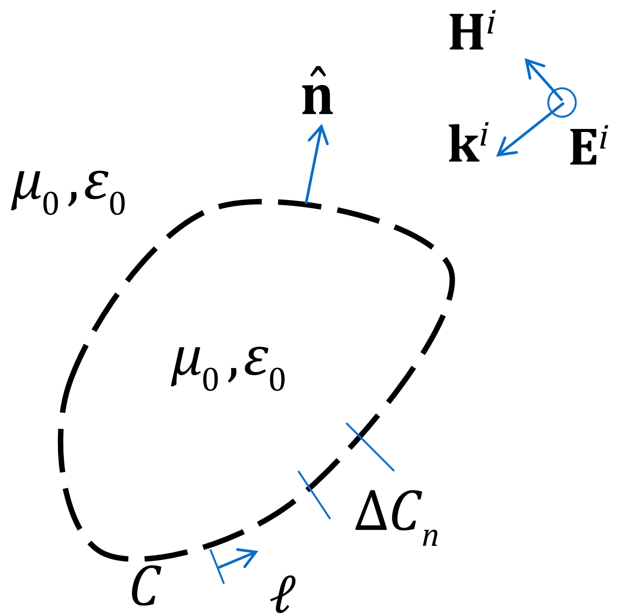



For TE polarization, the current has no axial component. Notice that a TE polarized wave can be also generated by an magnetic line source.

The MoM Procedure

# Expansion using Vector Basis Functions

$$\boldsymbol{\Lambda}_n(\boldsymbol{\rho}) = \begin{cases} \hat{\mathbf{l}}_n\left(\dfrac{l_n}{\Delta l_n}+1\right), & \boldsymbol{\rho}\in\Delta C_n \\ \hat{\mathbf{l}}_{n+1}\left(1-\dfrac{l_n}{\Delta l_{n+1}}\right), & \boldsymbol{\rho}\in\Delta C_{n+1} \\ 0, & \text{otherwise} \end{cases}$$




$$\mathbf{t}_n(\boldsymbol{\rho}) = \begin{cases} \hat{\mathbf{l}}_n, & \boldsymbol{\rho}\in\Delta C_n^{(2)} \\ \hat{\mathbf{l}}_{n+1}, & \boldsymbol{\rho}\in\Delta C_{n+1}^{(1)} \\ 0, & \text{otherwise} \end{cases}$$

For basis functions that span multiple segments, there should be special treatment for closed objects to account for the last unknown. (Explain)

Sample Results

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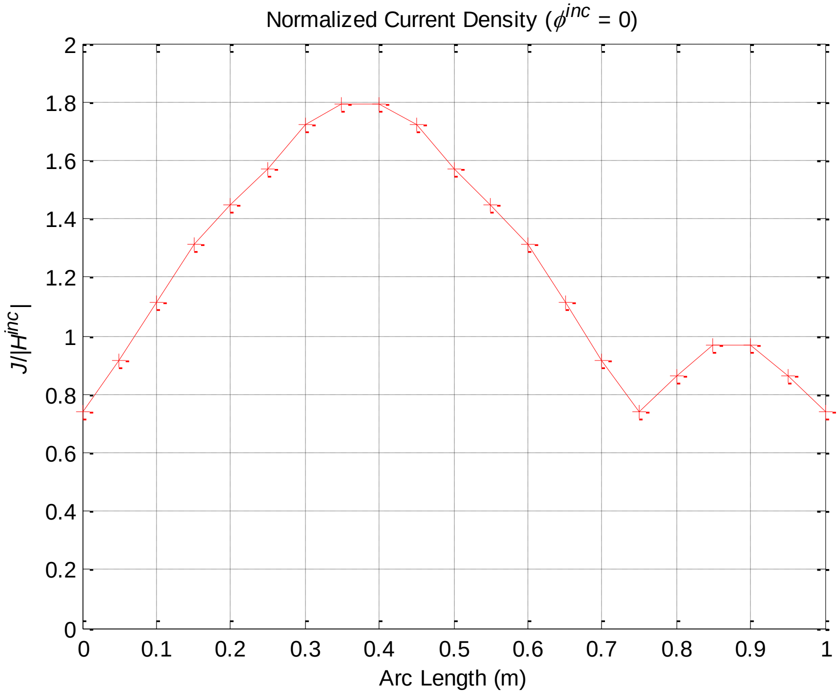


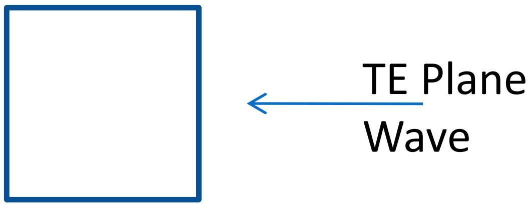


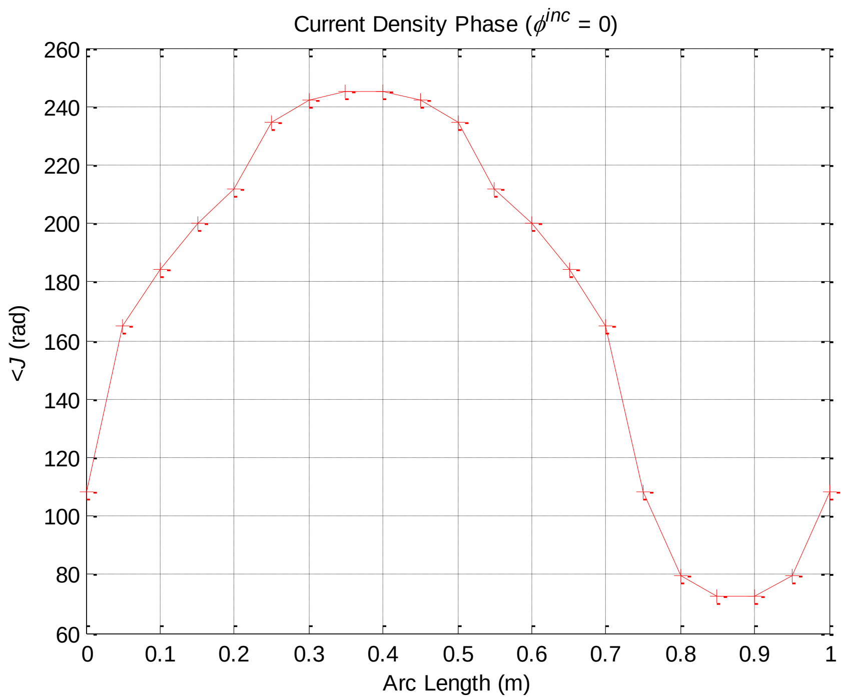


$f$ = 300 MHz

$\lambda$ = 1 m

Assignment

# Assignment #5

Repeat Assignment #3 using MFIE formulation.

# Assignment #6

Repeat Assignment #5 for TE polarization.

# Conclusion

- MFIE formulation is obtained by enforcing the boundary condition on the discontinuous field component.

- The MFIE does not have singular kernels and thus needs no special treatment for self-terms.

- Comparing the EFIE and the MFIE shows that the latter yields a "better" matrix equation. However, the EFIE can handle both open and closed objects.