Introduction to Computational Electromagnetics
Lecture 1: Introduction

ELC 657 – Spring 2017

Department of Electronics and Communications Engineering

Faculty of Engineering – Cairo University

Outline

1 CEM Techniques

- Classification
- Comparison and Limitations
- Challenges and Recent Advances

2 The Surface Equivalence Principle

- Historical Background
- Surface Integral Equation (SIE) Formulation

3 The Green's Function Method

- Typical Solution Procedure
- Known Green's Functions

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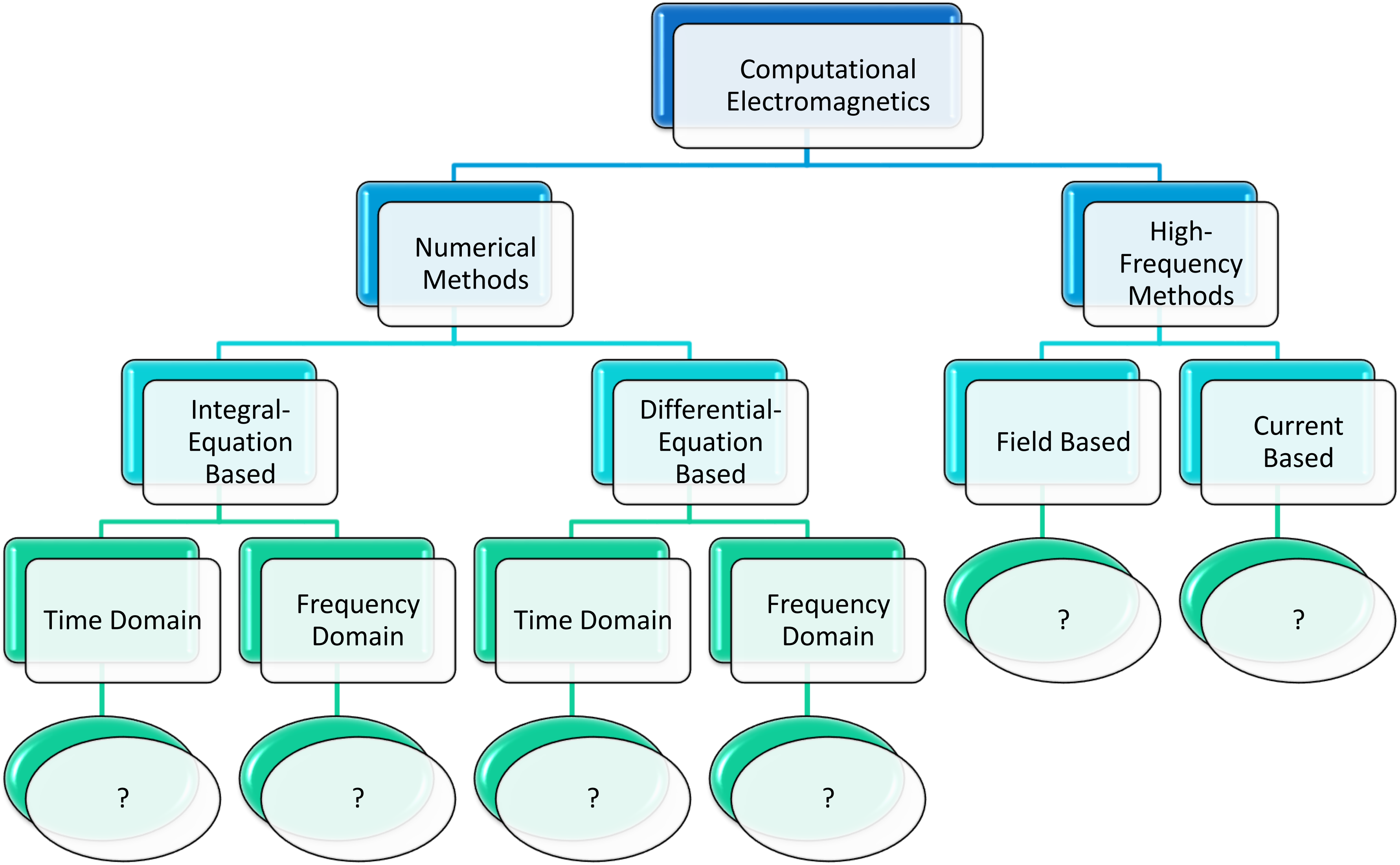
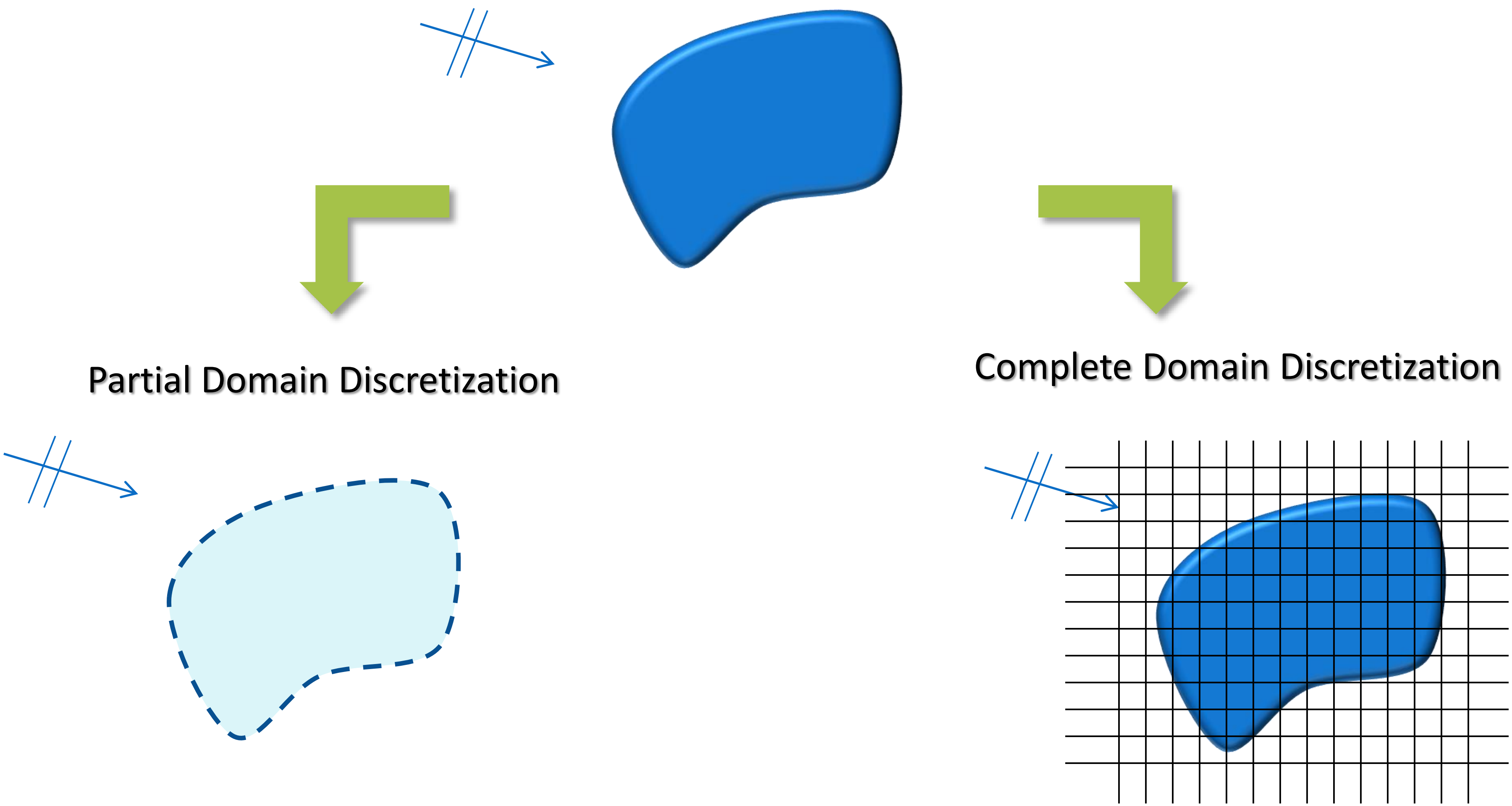
Classification

Classification Tree of CEM Techniques



Comparison and Limitations

Domain Discretization and Truncation and Memory Requirements



Comparison and Limitations

Time vs. Frequency Domain

Time Domain

Frequency Domain

Some Challenges in CEM

- Large objects
- Distant objects
- Adaptive meshing/ Sub-meshing
- Modeling new/artificial media
- Using GPU cards
- Optimization techniques

This Course

Course Topics

- Introduction to CEM
- Using the MoM! (2D PEC Electrostatic Problem)
- Background Theory (Projection Theorem)
- The MoM Procedure
- The MoM applied to 2D Electromagnetic Scattering Problem
  (EFIE – MFIE – CFIE)

- Introduction to Finite Differences
- Using the FD Technique!
- Applications to Electrostatic Problems, Transmission Lines… etc.

This Course

Grade Distribution

- Assignments/Presentations/…etc: 40%
- Projects: 20%
  - Project 1: General 2D PEC Electrostatic Problem
  - Project 2: General 2D PEC Electromagnetic Problem
  - Project 3: FD Solver
- Final: 40%

** The projects should be written in Matlab, C, or Fortran.*

This Course

References

- R. F. Harrington, *Field Computation by Moment Methods*. IEEE Press, 1993.
- A. W. Glisson and Atef Elsherbeni, Lecture Notes of ELE 528 and 628. The University of Mississippi, 2002-2003.
- C. A. Balanis, *Advanced Engineering Electromagnetics*. John Wiley and Sons, 1989.
- Selected papers.

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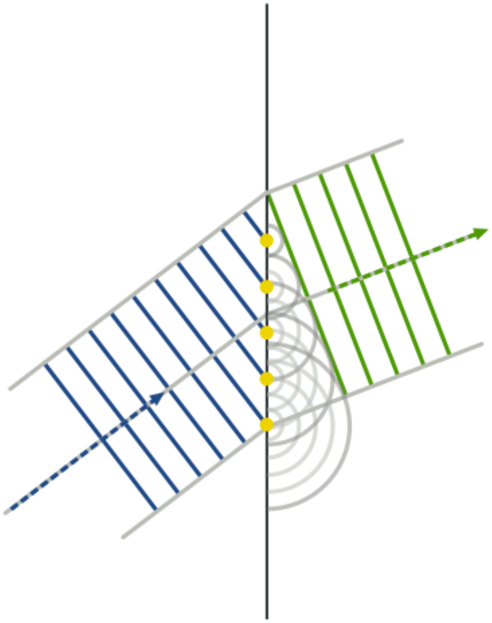
Historical Background

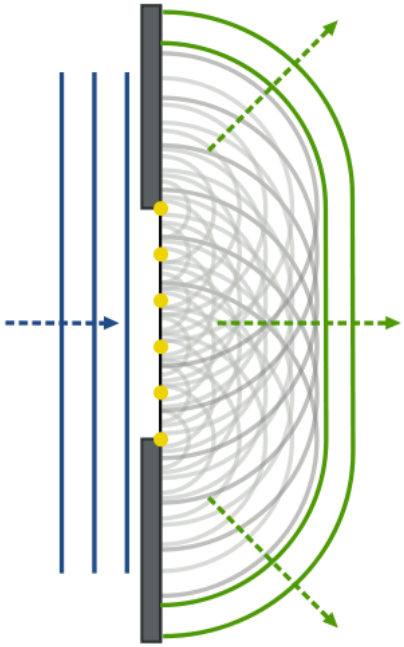
Huygens's Principle

Christiaan Huygens
(1629-1695)

Each point on the wavefront of an advancing wave (produced by some primary source) is the source of a new, secondary spherical wave.



Refraction in the manner of Huygens'



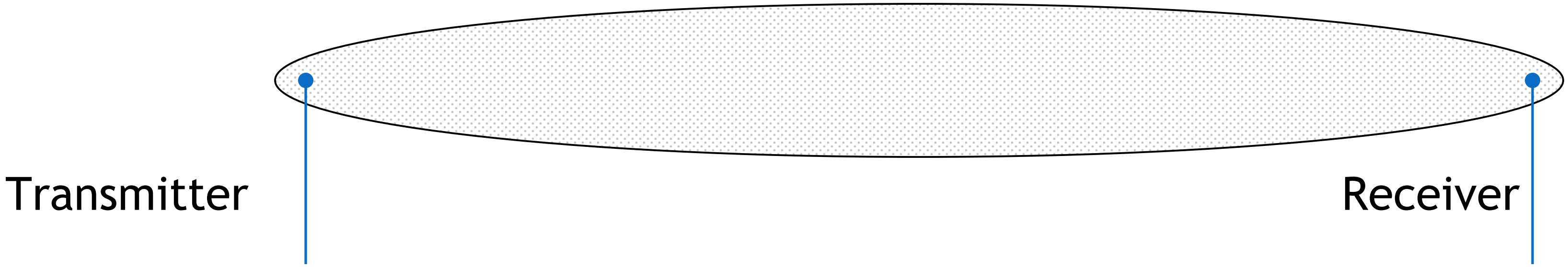
Diffraction in the manner of Huygens'

Historical Background

Fresnel Zones

Augustin-Jean Fresnel
(1788-1827)

Only a specific part of the wavefront has a significant contribution to the advancing wave, viz. the first Fresnel zone.



Historical Background

Balanced Maxwell's Equations

James Clerk Maxwell
(1831-1879)

$$\nabla\times\mathbf{E} = -j\omega\mu\mathbf{H}$$
$$\nabla\times\mathbf{H} = +j\omega\varepsilon\mathbf{E} + \mathbf{J}$$
$$\nabla\cdot\mathbf{D} = \rho$$
$$\nabla\cdot\mathbf{B} = 0$$

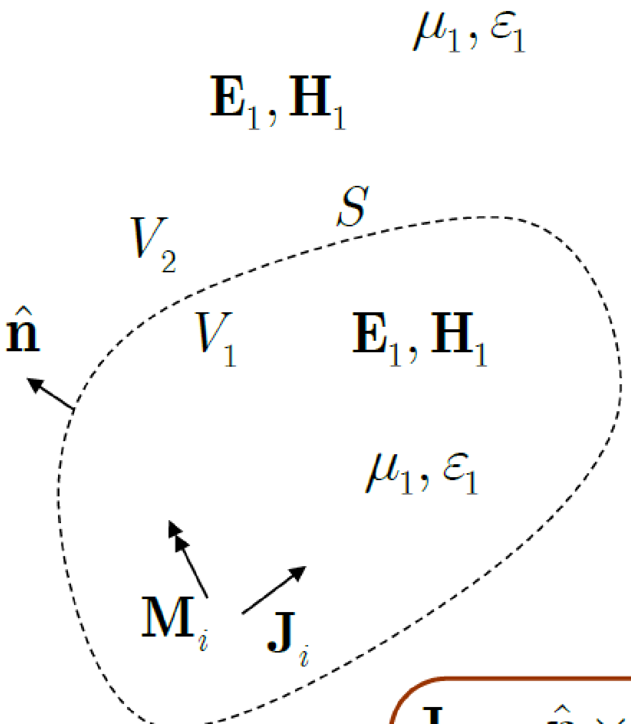
$$\nabla\times\mathbf{E} = -j\omega\mu\mathbf{H} - \mathbf{M}$$
$$\nabla\times\mathbf{H} = +j\omega\varepsilon\mathbf{E} + \mathbf{J}$$
$$\nabla\cdot\mathbf{D} = \rho_e$$
$$\nabla\cdot\mathbf{B} = \rho_m$$

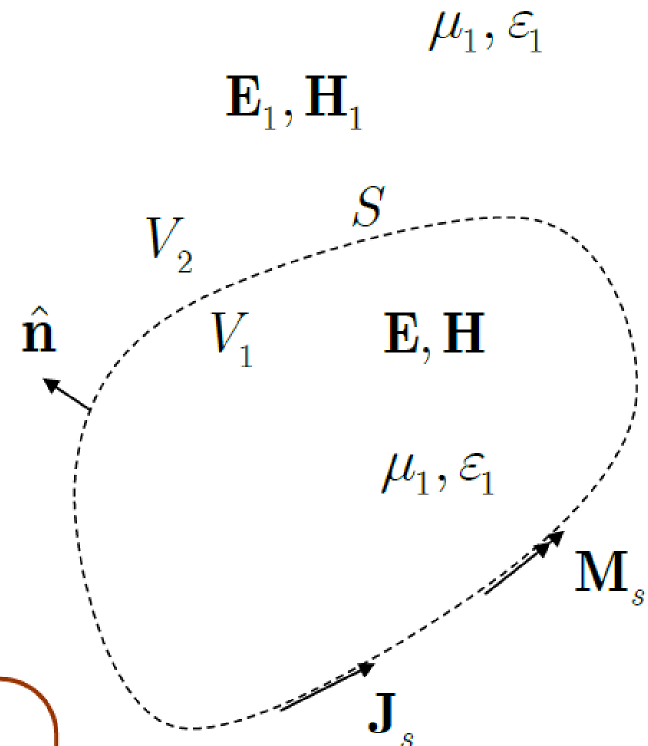
$$\mathbf{E} = \overbrace{-j\omega\mathbf{A} - \nabla\Phi}^{\mathbf{E}^J}\overbrace{-\frac{1}{\varepsilon}\nabla\times\mathbf{F}}^{\mathbf{E}^M}$$

$$\mathbf{H} = \underbrace{-j\omega\mathbf{F} - \nabla\Psi}_{\mathbf{H}^M} + \underbrace{\frac{1}{\mu}\nabla\times\mathbf{A}}_{\mathbf{H}^J}$$

Historical Background

Schelkunoff's Equivalence Principle



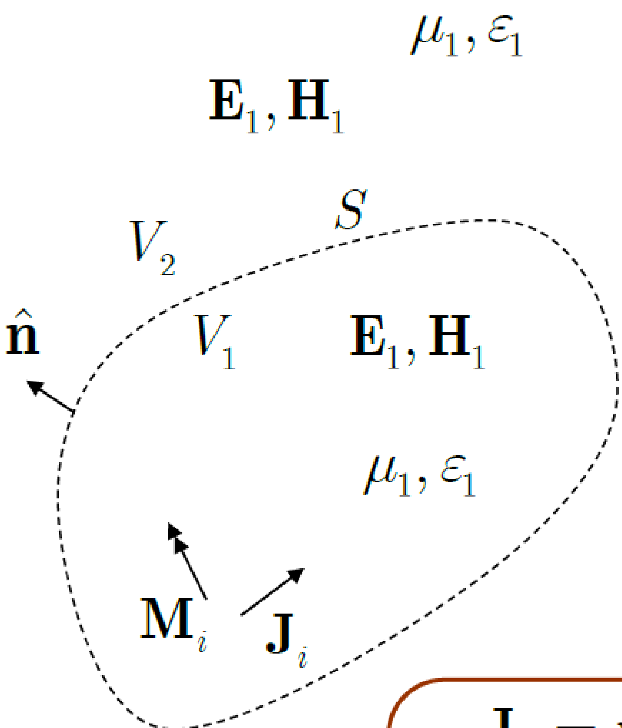


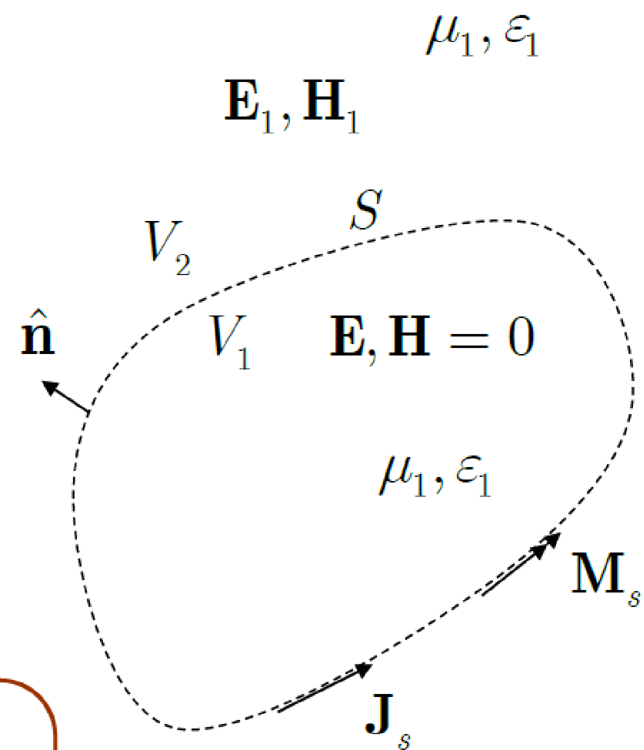
$$\boxed{\left.\begin{aligned} \mathbf{J}_s &= \hat{\mathbf{n}} \times (\mathbf{H}_1 - \mathbf{H}) \\ \mathbf{M}_s &= -\hat{\mathbf{n}} \times (\mathbf{E}_1 - \mathbf{E}) \end{aligned}\right\} \text{ on } S}$$

In Schelkunoff's equivalence, both fields need to be known on the surface, which is a redundancy according to the uniqueness theorem.

Historical Background

Love's Equivalence Principle





$$\left.\begin{aligned} \mathbf{J}_s &= \hat{\mathbf{n}} \times \mathbf{H}_1 \\ \mathbf{M}_s &= -\hat{\mathbf{n}} \times \mathbf{E}_1 \end{aligned}\right\} \text{on } S$$

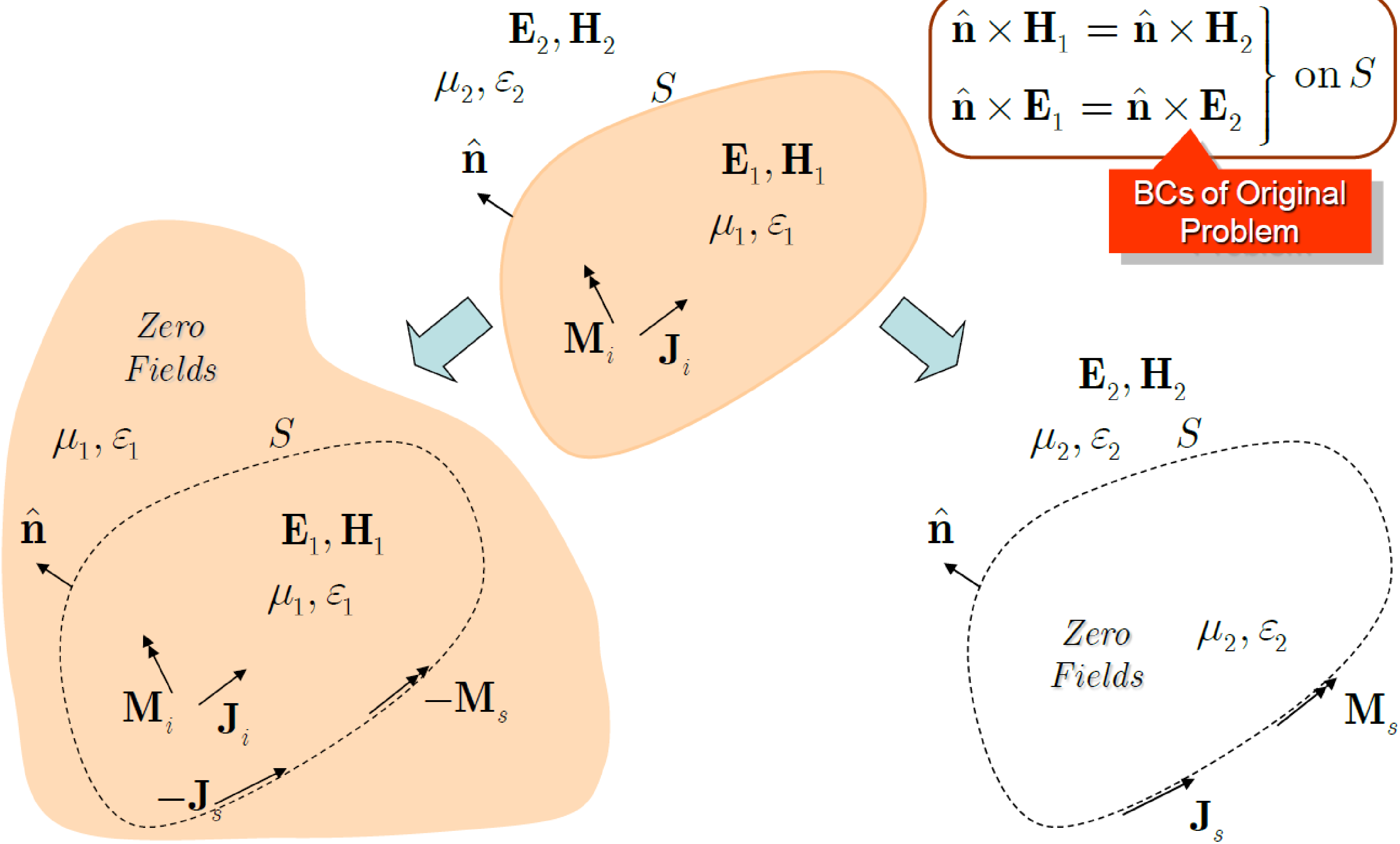
Notice that in the SE, the interior volume (V_1) can be filled with any homogeneous medium or perfectly conducting medium. But…?

Surface Integral Equation (SIE) Formulation

Equivalent Problems

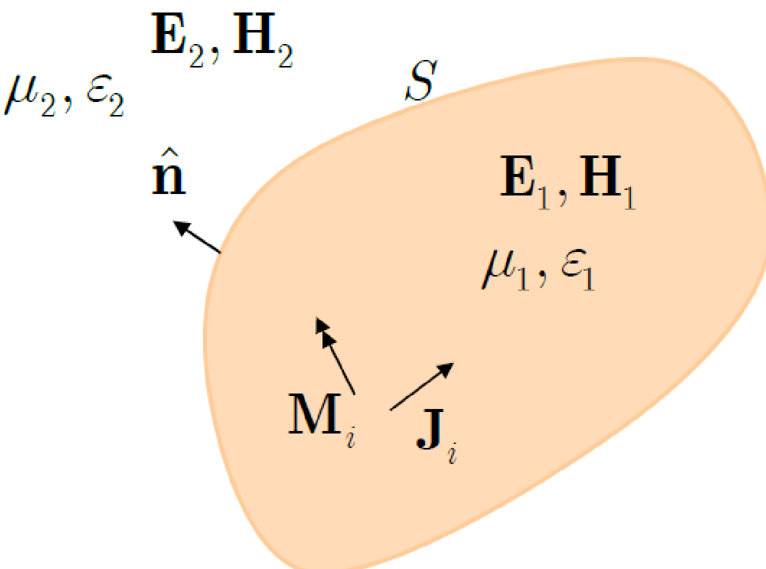
$$\left.\begin{aligned}\hat{\mathbf{n}}\times\mathbf{H}_1 &= \hat{\mathbf{n}}\times\mathbf{H}_2 \\ \hat{\mathbf{n}}\times\mathbf{E}_1 &= \hat{\mathbf{n}}\times\mathbf{E}_2\end{aligned}\right\} \text{ on } S$$

BCs of Original Problem



Surface Integral Equation (SIE) Formulation

Surface Integral Equations



$$\left.\begin{aligned} \hat{\mathbf{n}} \times \mathbf{H}_1 &= \hat{\mathbf{n}} \times \mathbf{H}_2 \\ \hat{\mathbf{n}} \times \mathbf{E}_1 &= \hat{\mathbf{n}} \times \mathbf{E}_2 \end{aligned}\right\} \text{on } S$$

Boundary Conditions in "Operator" Form

$$\left.\begin{aligned} &\hat{\mathbf{n}} \times \left[\mathbf{E}_1^J(\mathbf{J}_i) + \mathbf{E}_1^M(\mathbf{M}_i) + \mathbf{E}_1^J(-\mathbf{J}_s) + \mathbf{E}_1^M(-\mathbf{M}_s)\right] \\ &\qquad = \hat{\mathbf{n}} \times \left[\mathbf{E}_2^J(\mathbf{J}_s) + \mathbf{E}_2^M(\mathbf{M}_s)\right] \\ &\hat{\mathbf{n}} \times \left[\mathbf{H}_1^J(\mathbf{J}_i) + \mathbf{H}_1^M(\mathbf{M}_i) + \mathbf{H}_1^J(-\mathbf{J}_s) + \mathbf{H}_1^M(-\mathbf{M}_s)\right] \\ &\qquad = \hat{\mathbf{n}} \times \left[\mathbf{H}_2^J(\mathbf{J}_s) + \mathbf{H}_2^M(\mathbf{M}_s)\right] \end{aligned}\right\} \text{on } S$$

Surface Integral Equation (SIE) Formulation

The Fields as Operators

Fields due to J

$$\mathbf{E}_\nu^J(\mathbf{J}) = -j\omega\mathbf{A}_\nu(\mathbf{J}) - \nabla\Phi_\nu(\mathbf{J})$$

$$\mathbf{H}_\nu^J(\mathbf{J}) = \frac{1}{\mu_\nu}\nabla\times\mathbf{A}(\mathbf{J})$$

Fields due to M

$$\mathbf{H}_\nu^M(\mathbf{M}) = -j\omega\mathbf{F}_\nu(\mathbf{M}) - \nabla\Psi_\nu(\mathbf{M})$$

$$\mathbf{E}_\nu^M(\mathbf{M}) = -\frac{1}{\varepsilon_\nu}\nabla\times\mathbf{F}_\nu(\mathbf{M})$$

$$\mathbf{A}_\nu(\mathbf{J}) = \mu_\nu\iiint_V g_\nu(\mathbf{r},\mathbf{r}')\mathbf{J}(\mathbf{r}')dv'$$

$$\mathbf{F}_\nu(\mathbf{M}) = \varepsilon_\nu\iiint_V g_\nu(\mathbf{r},\mathbf{r}')\mathbf{M}(\mathbf{r}')dv'$$

$$\Phi_\nu(\mathbf{J}) = \frac{j}{\omega\varepsilon_\nu}\iiint_V g_\nu(\mathbf{r},\mathbf{r}')\nabla'\cdot\mathbf{J}(\mathbf{r}')dv'$$

$$\Psi_\nu(\mathbf{M}) = \frac{j}{\omega\mu_\nu}\iiint_V g_\nu(\mathbf{r},\mathbf{r}')\nabla'\cdot\mathbf{M}(\mathbf{r}')dv'$$

Free-Space Green's Function

$$g_\nu(\mathbf{r},\mathbf{r}') = \frac{e^{-jk_\nu|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

$$k_\nu = \omega\sqrt{\mu_\nu\varepsilon_\nu}$$

Surface Integral Equation (SIE) Formulation

The SIE Formulation and the MoM

The MoM is widely used in ***Surface Integral Equation*** formulations, where unknowns are placed only over the surfaces/boundaries of homogeneous regions.

- ☞ No need to discretize all space
- ☞ Reduced number of unknowns required

The MoM is easily applied to objects in unbounded regions.

- ☞ The radiation condition is automatically satisfied
- ☞ No "absorbing boundary" is required

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Typical Solution Procedure

Solution in Terms of Green's Function: 3D EM Problem

$$\left(\nabla^2 + k^2\right)\psi(\mathbf{r}) = s(\mathbf{r}) \qquad \left(\nabla^2 + k^2\right)g(\mathbf{r},\mathbf{r}') = -\delta(\mathbf{r}-\mathbf{r}')$$

$$\times g \qquad \times(-\psi)$$

$$+$$

$$g(\mathbf{r},\mathbf{r}')\nabla^2\psi(\mathbf{r}) - \psi(\mathbf{r})\nabla^2 g(\mathbf{r},\mathbf{r}') = g(\mathbf{r},\mathbf{r}')s(\mathbf{r}) + \psi(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')$$

$$\int_V \left\{g(\mathbf{r},\mathbf{r}')\nabla^2\psi(\mathbf{r}) - \psi(\mathbf{r})\nabla^2 g(\mathbf{r},\mathbf{r}')\right\}dv = \int_V g(\mathbf{r},\mathbf{r}')s(\mathbf{r})dv + \psi(\mathbf{r}')$$

$$\int_S \left\{g(\mathbf{r},\mathbf{r}')\frac{\partial\psi(\mathbf{r})}{\partial n} - \psi(\mathbf{r})\frac{\partial g(\mathbf{r},\mathbf{r}')}{\partial n}\right\}ds = \int_V g(\mathbf{r},\mathbf{r}')s(\mathbf{r})dv + \psi(\mathbf{r}')$$

Green's Identity

$$\psi(\mathbf{r}') = -\int_V g(\mathbf{r},\mathbf{r}')s(\mathbf{r})dv$$

$$\psi(\mathbf{r}) = -\int_V g(\mathbf{r},\mathbf{r}')s(\mathbf{r}')dv$$

Typical Solution Procedure

Determining the Green's Function: 3D EM Problem

$$\left(\nabla^2+k^2\right)g\left(\mathbf{r},\mathbf{r}'\right)=-\delta\left(\mathbf{r}-\mathbf{r}'\right)$$

Source at Origin

$$\left(\nabla^2+k^2\right)g\left(r\right)=-\delta\left(r\right)$$

$$\int_V$$

$$\int_V \nabla^2 g\left(r\right)dv+k^2\int_V g\left(r\right)dv=-1$$

$$\int_S \nabla g\left(r\right)\cdot d\mathbf{s}+k^2\int_V g\left(r\right)dv=-1$$

$$\lim_{a\to 0}\int_S \nabla g\left(r\right)\cdot d\mathbf{s}+k^2\lim_{a\to 0}\cancel{\int_V g\left(r\right)dv}=-1$$

$$\left(\nabla^2+k^2\right)g\left(r\right)=0,\; r\neq 0$$

$$g\left(r\right)=A\frac{e^{-jkr}}{r}$$

$$\lim_{a\to 0}\int_S A\frac{-e^{-jka}\left(jka+1\right)}{a^2}a^2\sin\theta\, d\theta\, d\varphi=-1$$

$$A=\frac{1}{4\pi}$$

$$\boxed{g\left(\mathbf{r},\mathbf{r}'\right)=\frac{e^{-jk\left|\mathbf{r}-\mathbf{r}'\right|}}{4\pi\left|\mathbf{r}-\mathbf{r}'\right|}}$$

Known Green's Functions

Examples of Known Green's Functions

$$g_{3D}\left(\mathbf{r},\mathbf{r}'\right)=\frac{e^{-jk\left|\mathbf{r}-\mathbf{r}'\right|}}{4\pi\left|\mathbf{r}-\mathbf{r}'\right|}$$

$$g_{2D}\left(\boldsymbol{\rho},\boldsymbol{\rho}'\right)=\frac{1}{4j}H_0^{(2)}\left(k\left|\boldsymbol{\rho}-\boldsymbol{\rho}'\right|\right)$$

$$g_{1D}\left(z,z'\right)=\frac{e^{-jk\left|z-z'\right|}}{2jk}$$

Other known Green's functions include half-space, waveguide, parallel plate, and layered media Green's functions.

- CEM includes several techniques, each is fit for certain problems and has its advantages and disadvantages.
- The surface equivalence principle offers a solution to EM scattering problems in terms of a set of equivalent problems and results in surface integral equations (that can be solved using the MoM).
- The Green's function is the "spatial" impulse response of the system.