Assignment (2)

1

$$A = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$$

$$X_1^0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \end{bmatrix} \quad \lambda_1^1 = 8 \quad X_1^1 = \begin{bmatrix} 1 \\ 0.625 \end{bmatrix}$$

Repeat $\rightsquigarrow$

$$\lambda_1^2 = 7.25 \quad X_1^2 = \begin{bmatrix} 1 \\ 0.5345 \end{bmatrix}$$

$$\lambda_1^3 = 7.07 \quad X_1^3 = \begin{bmatrix} 1 \\ 0.5098 \end{bmatrix}$$

$$\lambda_1^4 = 7.02 \quad X_1^4 = \begin{bmatrix} 1 \\ 0.50278 \end{bmatrix}$$

$$\lambda_1^5 = 7 \qquad X_1^5 = \begin{bmatrix} 1 \\ 0.5008 \end{bmatrix}$$

↳ Repeat

$$\therefore \quad \lambda_1 = 7 \qquad X_1 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{18-4} \begin{bmatrix} 3 & -2 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 0.214 & -0.143 \\ -0.143 & 0.429 \end{bmatrix}$$

assume $X_2^0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 0.214 & -0.143 \\ -0.143 & 0.429 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.07 \\ 0.286 \end{bmatrix}$$

$$\lambda_{2\,inv}^{-1} = 0.286 \qquad X_2^1 = \begin{bmatrix} 0.248 \\ 1 \end{bmatrix}$$

Repeat: $\lambda_2^{-1} = 0.5$ $\qquad X_2 = \begin{bmatrix} -0.24 \\ 0.5 \end{bmatrix}$

↳ $\lambda_2 = 2$

2) $A = \begin{bmatrix} 7 & 4 & 1 \\ 4 & 4 & 4 \\ 1 & 4 & 7 \end{bmatrix}$

assume $X_1^0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 7 & 4 & 1 \\ 4 & 4 & 4 \\ 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \\ 12 \end{bmatrix} \quad \lambda_1^1 = 12 \quad X_1^1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore \lambda_1 = 12 \quad X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

normalized $X_1 = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} = \begin{bmatrix} 0.5774 \\ 0.5774 \\ 0.5774 \end{bmatrix}$

$$B = \begin{bmatrix} 7 & 4 & 1 \\ 4 & 4 & 4 \\ 1 & 4 & 7 \end{bmatrix} - 12 \begin{bmatrix} 0.5774 \\ 0.5774 \\ 0.5774 \end{bmatrix} \begin{bmatrix} 0.5774 & 0.5774 & 0.5774 \end{bmatrix} = \begin{bmatrix} 3 & 0 & -3 \\ 0 & 0 & 0 \\ -3 & 0 & 3 \end{bmatrix}$$

Assume $X_2^0 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$$\begin{bmatrix} 3 & 0 & -3 \\ 0 & 0 & 0 \\ -3 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ -6 \end{bmatrix} \quad \lambda_2^1 = 6 \quad X_2^1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\therefore \lambda_2 = 6 \qquad X_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Normalize $X_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -1/\sqrt{2} \end{bmatrix}$

$$C = \begin{bmatrix} 3 & 0 & -3 \\ 0 & 0 & 0 \\ -3 & 0 & 3 \end{bmatrix} - 6 \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightsquigarrow \therefore \lambda_3 = 0$$

To get the corresponding eigen vector, refer to the first definition of eigen value problem:

$$[A - \lambda I] X = 0$$

$$\left\{ \begin{bmatrix} 7 & 4 & 1 \\ 4 & 4 & 4 \\ 1 & 4 & 7 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$7X_1 + 4X_2 + X_3 = 0$$

$$4X_1 + 4X_2 + 4X_3 = 0$$

$$X_1 + 4X_2 + 7X_3 = 0$$

assume $X_1 = 1$:

$$7 + 4X_2 + X_3 = 0 \Rightarrow X_3 = -7 - 4X_2$$

$$1 + X_2 + X_3 = 0 \Rightarrow X_3 = -1 - X_2$$

$$1 + 4X_2 + 7X_3 = 0 \Rightarrow X_3 = \frac{-1 - 4X_2}{7}$$

Solving : the eigen vector $X_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ $\lambda_3 = 0$

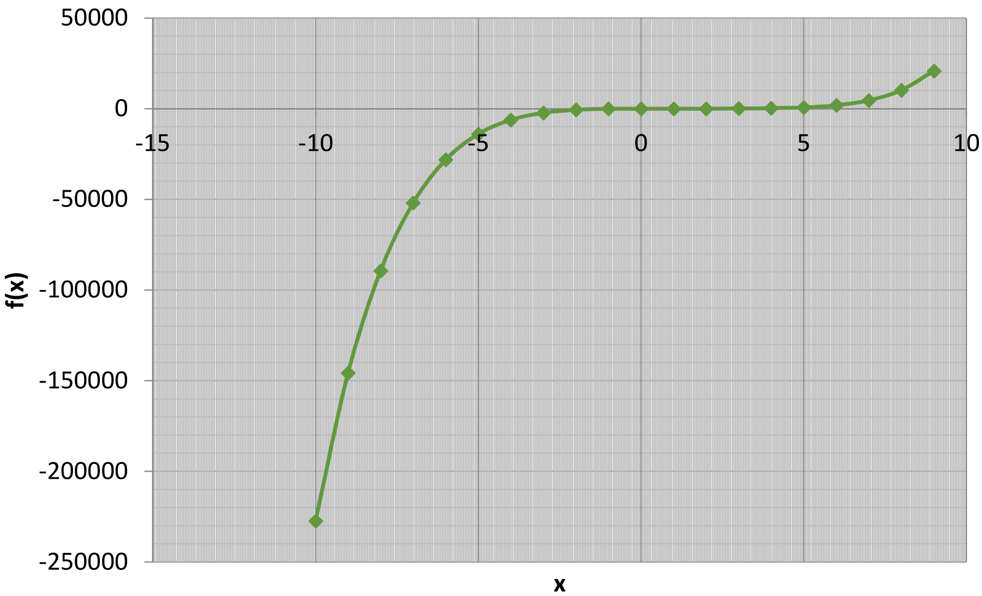
Assignment 2

3- For the equation: $f(x) = 10 + 20x - 42x^2 + 33x^3 - 9x^4 + x^5$

   i. Determine all the roots graphically.

   ii. Use bisection and false position methods to determine the root to $\epsilon_a = 10\%$. Employ initial guesses of $x_l = 0.5$ and $x_u = 1$.

a. Graphically: x = -0.2862



b. Using the bisection method

x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	$f(x_r)f(x_r)$	condition	error	
0.5	13.0938	1	13.0000	0.75	2.83105	37.0691	xl=xr	--	
0.75	2.83105	1	5.0000	0.875	4.00314	11.3331	xl=xr	14.3%	continue
0.875	4.00314	1	5.0000	0.9375	4.5063	18.0394	xl=xr	6.7%	stop iteration
0.9375	4.5063	1	5.0000	0.96875	4.75226	21.4151	xl=xr	3.2%	stop iteration
0.96875	4.75226	1	5.0000	0.98438	4.87565	23.1703	xl=xr	1.6%	stop iteration

Comment: Solution converged to a wrong solution with error < 10%

4- Determine the positive real root of ln $(x^4) = 0.7$:

i. Analytically.

ii. Graphically.

iii. Using three iterations of the false-position and bisection methods with initial guesses of 0.5 and 2. Compute the approximate error ε_a and the true error ε_t after each iteration. Comment on the results.

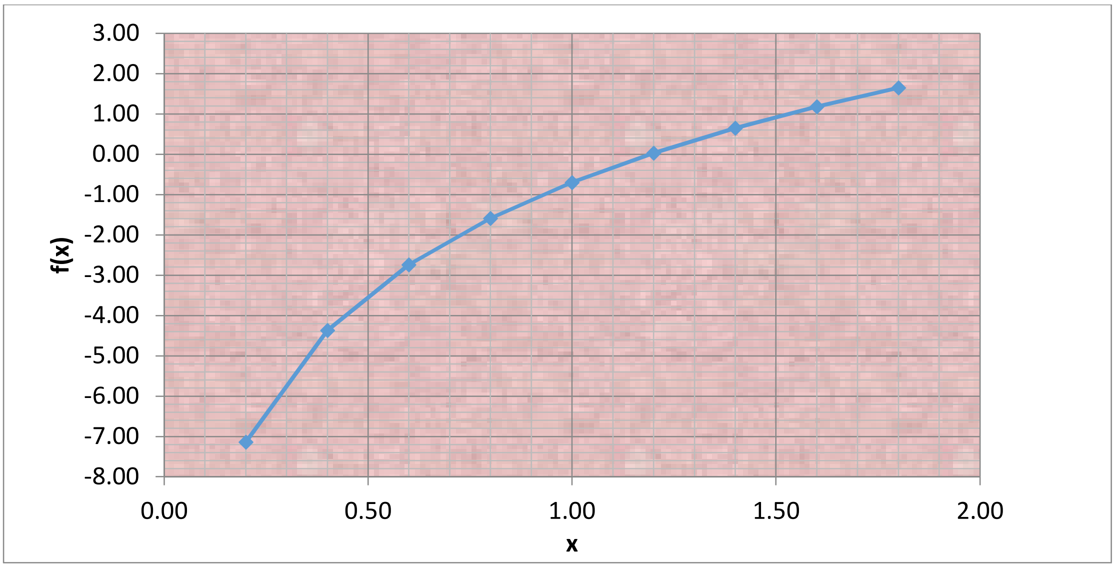
i. Analytically:

$x^4 = e^{0.7}$

$x = 1.1912$

ii. Graphically:

$x = 1.1912$



iii. False Position Iterations:

Iteration	x_l	$f(x_l)$	x_u	$f(x_u)$	x_r	$f(x_r)$	$f(x_r)f(x_r)$	condition	Approx. error	True error
1	0.5	-3.4726	2	2.07259	1.43935	0.75678	-2.628	xu=xr	--	--
2	0.5	-3.4726	1.43935	0.75678	1.27127	0.26007	-0.9031	xu=xr	13.2%	6.8%
3	0.5	-3.4726	1.27127	0.26007	1.21753	0.08731	-0.3032	xu=xr	4.4%	2.3%

5- Determine a root of f (x) = - x^2 + 1.8 x + 2.5 :

i. Graphically.

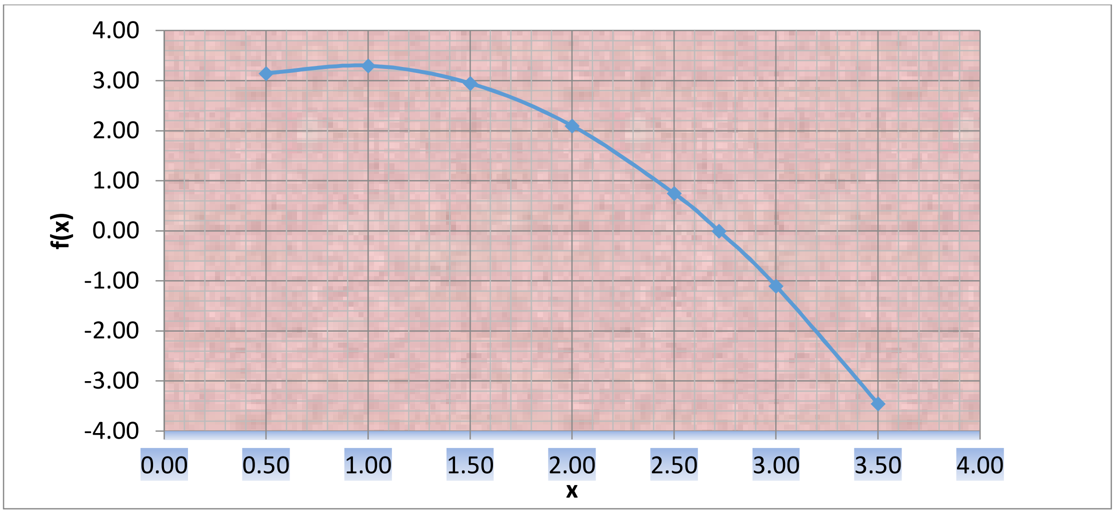
ii. Fixed-point iteration method.

iii. Newton-Raphson method.

Use $x_0 = 5$, perform computations until ϵ_a is less than 0.05%.

i. Graphically:

x = 2.72



ii. Fixed Point Iteration Method:

g(x)=xi =(1.8x+2.5)^.5

g′(x)=1.8*.5(1.8*x+2.5)^-.5

Iteration	x_i	$g'(x_i)$	Approx. error	
0	5	0.2654	--	converging
1	3.39116	0.30682	47.4%	converging
2	2.93327	0.32267	15.6%	converging
3	2.78925	0.32818	5.2%	converging
4	2.74238	0.33004	1.7%	converging
5	2.72695	0.33066	0.6%	converging
6	2.72186	0.33086	0.2%	converging
7	2.72017	0.33093	0.1%	converging
8	2.71962	0.33095	0.0%	converging

iii. Newton-Raphson Method:

$f(x) = -x^2+1.8x+2.5$

$f'(x) = -2x+1.8$

Iteration	x_i	Approx. error	
0	5	--	
1	3.35365854	49.1%	continue
2	2.80133225	19.7%	continue
3	2.72110842	2.9%	continue
4	2.7193414	0.1%	continue
5	2.71934054	0.0%	stop iteration

6- Locate the first positive root of $f(x) = \sin x + \cos(1 + x^2) - 1$ using four iterations of the Secant method with initial guesses:

i. $x_i^{-1} = 1.0$ and $x_i = 3.0$.

ii. $x_i^{-1} = 1.5$ and $x_i = 2.5$.

iii. $x_i^{-1} = 1.5$ and $x_i = 2.25$.

Use the graphical method to explain your results.

i. Secant Method: $x_i^{-1} = 1.0$ and $x_i = 3.0$.

$f(x) = \sin x+\cos(1+x^2)-1$

$x_{i-1} = 1$	$xi = 3$

Iteration	x_{i-1}	$f(x_{i-1})$	x_i	$f(x_i)$	x_{i+1}	Approx. error
1	1	-0.57467585	3	-1.698	-0.0232	13023.1%
2	3	-1.69795152	-0.0232	-0.4834	-1.2263	98.1%
3	-0.02321428	-0.48336344	-1.2263	-2.7448	0.23395	624.2%
4	-1.22634748	-2.74475001	0.23395	-0.2747	0.39637	41.0%

ii. Secant Method: $x_i^{-1} = 1.5$ and $x_i = 2.5$.

$f(x) = \sin x+\cos(1+x^2)-1$

x_{i-1} =1.5	xi =2.5

Iteration	x_{i-1}	$f(x_{i-1})$	x_i	$f(x_i)$	x_{i+1}	Approx. error
1	1.5	-0.99663469	2.5	0.1664	2.35693	6.1%
2	2.5	0.16639632	2.35693	0.66984	2.54729	7.5%
3	2.35692873	0.66984231	2.54729	-0.0828	2.52634	0.8%
4	2.54728716	-0.08282791	2.52634	0.03147	2.53211	0.2%

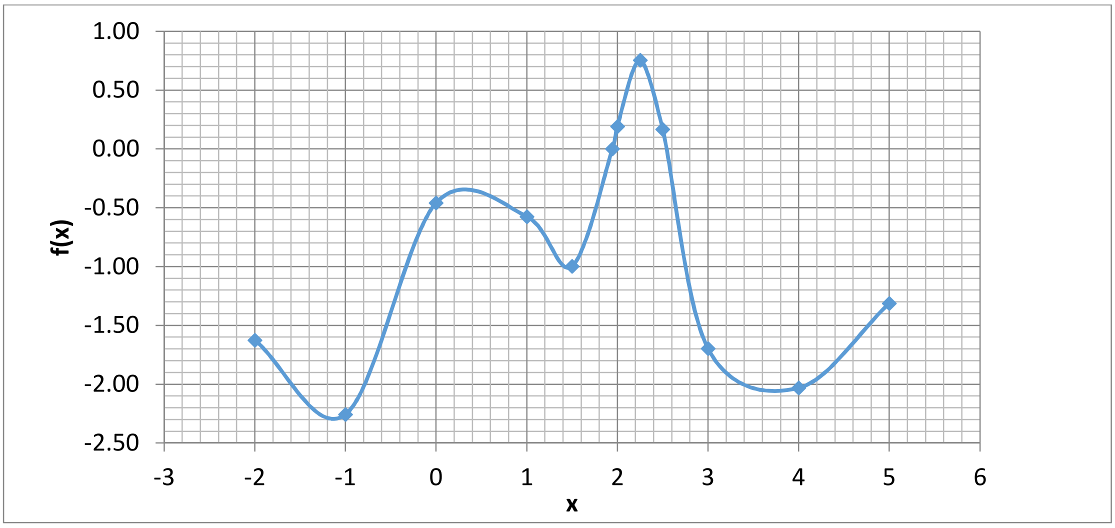
iii. Secant Method: $x_i^{-1} = 1.5$ and $x_i = 2.25$.

$f(x) = \sin x+\cos(1+x^2)-1$

x_{i-1} =1.5	xi =2.25

Iteration	x_{i-1}	$f(x_{i-1})$	x_i	$f(x_i)$	x_{i+1}	Approx. error
1	1.5	-0.99663469	2.25	0.75382	1.92702	16.8%
2	2.25	0.75382086	1.92702	-0.0618	1.95148	1.3%
3	1.92701799	-0.06176948	1.95148	0.02415	1.9446	0.4%
4	1.95147933	0.02414683	1.9446	-1E-05	1.94461	0.0%

iv. Graphical Solution



x = 1.94461

Comment: changing the bounding values resulted in changing the solution. In the first iteration, convergence was not attained after four trials while solution in all cases is between the selected range.

7- Determine the real root of $x^{3.5} = 80$, with the modified secant method to within $\epsilon_a = 0.1\%$ using an initial guess of $x_o = 3.5$ and $\delta = 0.01$.

$f(x) = x^{3.5} - 80$

$\delta =$ 0.01

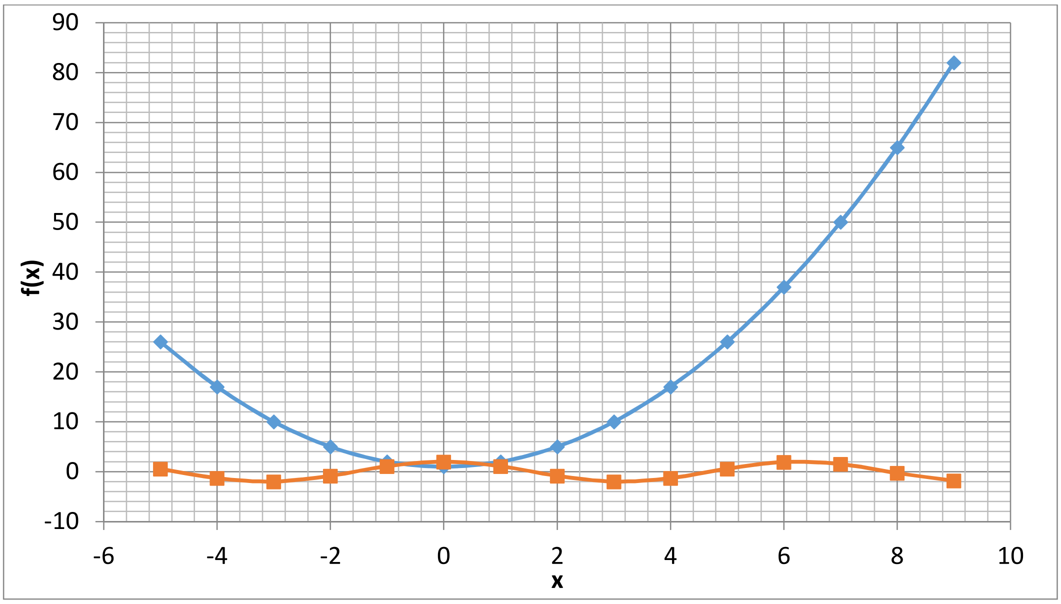
Iteration	x_i	$f(x_i)$	$x_i + \delta x_i$	$f(x_i+ \delta x_i)$	x_{i+1}	Approx. error
1	3.5	0.2117802	3.535	3.05446	3.49739	0.1%
2	3.4973925	0.0028222	3.53237	2.8381	3.49736	0.0%

8- Determine the roots of the following simultaneous nonlinear equations using (i) fixed-point iteration and (ii) the Newton-Raphson method:

$$y = x^2 + 1$$

$$y = 2 \cos x$$

Use a graphical approach to obtain initial guesses.



i. Fixed Point Iteration

$x = (y-1)^{0.5}$

$y = 2COSX$

Iteration	x	y	error (x)	error (y)	
0	-0.6000	1.5000	--	--	
1	0.7071	1.6507	184.9%	9.1%	converging
2	0.8066	1.5205	12.3%	8.6%	converging
3	0.7214	1.3839	11.8%	9.9%	converging
4	0.6196	1.5017	16.4%	7.8%	converging
5	0.7083	1.6283	12.5%	7.8%	converging
6	0.7926	1.5189	10.6%	7.2%	converging
7	0.7204	1.4039	10.0%	8.2%	converging
i8	0.6356	1.5031	13.3%	6.6%	converging
i9	0.7093	1.6095	10.4%	6.6%	converging
i10	0.7807	1.5176	9.1%	6.1%	converging
i11	0.7195	1.4209	8.5%	6.8%	converging
i12	0.6487	1.5043	10.9%	5.5%	converging
i13	0.7102	1.5937	8.6%	5.6%	converging
i14	0.7705	1.5165	7.8%	5.1%	converging
i15	0.7187	1.4351	7.2%	5.7%	converging
i16	0.6596	1.5053	9.0%	4.7%	converging
i17	0.7109	1.5804	7.2%	4.8%	converging
i18	0.7619	1.5156	6.7%	4.3%	converging
i19	0.7180	1.4471	6.1%	4.7%	converging
i20	0.6686	1.5062	7.4%	3.9%	converging
i21	0.7115	1.5693	6.0%	4.0%	converging
i22	0.7545	1.5148	5.7%	3.6%	converging
i23	0.7175	1.4572	5.2%	4.0%	converging
i24	0.6762	1.5069	6.1%	3.3%	converging
i25	0.7120	1.5600	5.0%	3.4%	converging
i26	0.7483	1.5142	4.9%	3.0%	converging
i27	0.7170	1.4657	4.4%	3.3%	converging
i28	0.6824	1.5075	5.1%	2.8%	converging
i29	0.7124	1.5521	4.2%	2.9%	converging
i30	0.7430	1.5136	4.1%	2.5%	converging
i31	0.7167	1.4728	3.7%	2.8%	converging
i32	0.6876	1.5080	4.2%	2.3%	converging
i33	0.7127	1.5455	3.5%	2.4%	converging
i34	0.7386	1.5131	3.5%	2.1%	converging
i35	0.7163	1.4788	3.1%	2.3%	converging
i36	0.6920	1.5084	3.5%	2.0%	converging
i37	0.7130	1.5400	3.0%	2.0%	converging
i38	0.7348	1.5127	3.0%	1.8%	converging
i39	0.7161	1.4839	2.6%	1.9%	converging
i40	0.6956	1.5088	2.9%	1.7%	converging

i41	0.7133	1.5353	2.5%	1.7%	converging
i42	0.7316	1.5124	2.5%	1.5%	converging
i43	0.7158	1.4882	2.2%	1.6%	converging
i44	0.6987	1.5091	2.5%	1.4%	converging
i45	0.7135	1.5314	2.1%	1.5%	converging
i46	0.7290	1.5121	2.1%	1.3%	converging
i47	0.7156	1.4917	1.9%	1.4%	converging
i48	0.7012	1.5093	2.1%	1.2%	converging
i49	0.7137	1.5281	1.7%	1.2%	converging
i50	0.7267	1.5119	1.8%	1.1%	converging
i51	0.7155	1.4947	1.6%	1.1%	converging
i52	0.7034	1.5096	1.7%	1.0%	converging
i53	0.7138	1.5253	1.5%	1.0%	converging
i54	0.7248	1.5117	1.5%	0.9%	converging
i55	0.7153	1.4973	1.3%	1.0%	converging
i56	0.7052	1.5097	1.4%	0.8%	converging
i57	0.7140	1.5230	1.2%	0.9%	converging
i58	0.7232	1.5116	1.3%	0.8%	converging
i59	0.7152	1.4994	1.1%	0.8%	converging
i60	0.7067	1.5099	1.2%	0.7%	converging
i61	0.7141	1.5210	1.0%	0.7%	converging
i62	0.7218	1.5114	1.1%	0.6%	converging
i63	0.7151	1.5012	0.9%	0.7%	converging
i64	0.7080	1.5100	1.0%	0.6%	converging
i65	0.7142	1.5194	0.9%	0.6%	converging
i66	0.7207	1.5113	0.9%	0.5%	converging
i67	0.7150	1.5027	0.8%	0.6%	converging
i68	0.7090	1.5101	0.9%	0.5%	converging
i69	0.7142	1.5180	0.7%	0.5%	converging
i70	0.7197	1.5112	0.8%	0.5%	converging
i71	0.7150	1.5040	0.7%	0.5%	converging
i72	0.7099	1.5102	0.7%	0.4%	converging
i73	0.7143	1.5168	0.6%	0.4%	converging
i74	0.7189	1.5111	0.6%	0.4%	converging
i75	0.7149	1.5050	0.6%	0.4%	converging
i76	0.7107	1.5103	0.6%	0.3%	converging
i77	0.7143	1.5159	0.5%	0.4%	converging
i78	0.7182	1.5110	0.5%	0.3%	converging
i79	0.7149	1.5059	0.5%	0.3%	converging
i80	0.7113	1.5103	0.5%	0.3%	converging
i81	0.7144	1.5150	0.4%	0.3%	converging
i82	0.7177	1.5110	0.5%	0.3%	converging

i83	0.7148	1.5067	0.4%	0.3%	converging
i84	0.7118	1.5104	0.4%	0.2%	converging
i85	0.7144	1.5143	0.4%	0.3%	converging
i86	0.7172	1.5109	0.4%	0.2%	converging
i87	0.7148	1.5073	0.3%	0.2%	converging
i88	0.7123	1.5104	0.4%	0.2%	converging
i89	0.7145	1.5138	0.3%	0.2%	converging
i90	0.7168	1.5109	0.3%	0.2%	converging
i91	0.7148	1.5079	0.3%	0.2%	converging
i92	0.7126	1.5105	0.3%	0.2%	converging
i93	0.7145	1.5133	0.3%	0.2%	converging
i94	0.7164	1.5109	0.3%	0.2%	converging
i95	0.7147	1.5083	0.2%	0.2%	converging
i96	0.7130	1.5105	0.3%	0.1%	converging
i97	0.7145	1.5129	0.2%	0.2%	converging
i98	0.7161	1.5108	0.2%	0.1%	converging
i99	0.7147	1.5087	0.2%	0.1%	converging
i100	0.7132	1.5105	0.2%	0.1%	converging
i101	0.7145	1.5125	0.2%	0.1%	converging
i102	0.7159	1.5108	0.2%	0.1%	converging
i103	0.7147	1.5090	0.2%	0.1%	converging
i104	0.7134	1.5106	0.2%	0.1%	converging
i105	0.7145	1.5122	0.2%	0.1%	converging
i106	0.7157	1.5108	0.2%	0.1%	converging
i107	0.7147	1.5093	0.1%	0.1%	converging
i108	0.7136	1.5106	0.1%	0.1%	converging
i109	0.7146	1.5120	0.1%	0.1%	converging
i110	0.7155	1.5108	0.1%	0.1%	converging
i111	0.7147	1.5095	0.1%	0.1%	converging
i112	0.7138	1.5106	0.1%	0.1%	converging
i113	0.7146	1.5118	0.1%	0.1%	converging
i114	0.7154	1.5108	0.1%	0.1%	converging
i115	0.7147	1.5097	0.1%	0.1%	converging
i116	0.7139	1.5106	0.1%	0.1%	converging
i117	0.7146	1.5116	0.1%	0.1%	converging

ii. Newton-Raphson Method

$u = X^2 + 1 - Y$

$\delta u/\delta x =$ 2x

$\delta u/\delta y =$ -1

$v = -y + 2COSX$

$\delta v/\delta x =$ -2sinx

$\delta v/\delta y =$ -1

Iteration	x	y	u	v	δu/δx	δu/δy	δv/δx	δv/δy	error (x)	error (y)
0	1	1	1	0.0806	2	-1	-1.6829	-1	--	--
1	0.75036	1.50073	0.06232	-0.0378	1.50073	-1	-1.3638	-1	33.3%	33.4%
2	0.7154	1.51057	0.00122	-0.0009	1.43079	-1	-1.3118	-1	4.9%	0.7%
3	0.71462	1.51068	6E-07	-5E-07	1.42924	-1	-1.3107	-1	0.1%	0.0%
4	0.71462	1.51068	1.5E-13	-1E-13	1.42924	-1	-1.3107	-1	0.0%	0.0%
5	0.71462	1.51068	0	0	1.42924	-1	-1.3107	-1	0.0%	0.0%

X=	0.71462
Y=	1.51068