CUFE, M. Sc., 2015-2016

Computers & Numerical Analysis (STR 681)

Lecture 10
OPTIMIZATION

Dr. Maha Moddather

Structural Engineering Department

Faculty of Engineering – Cairo University

mahamoddather@eng.cu.edu.eg

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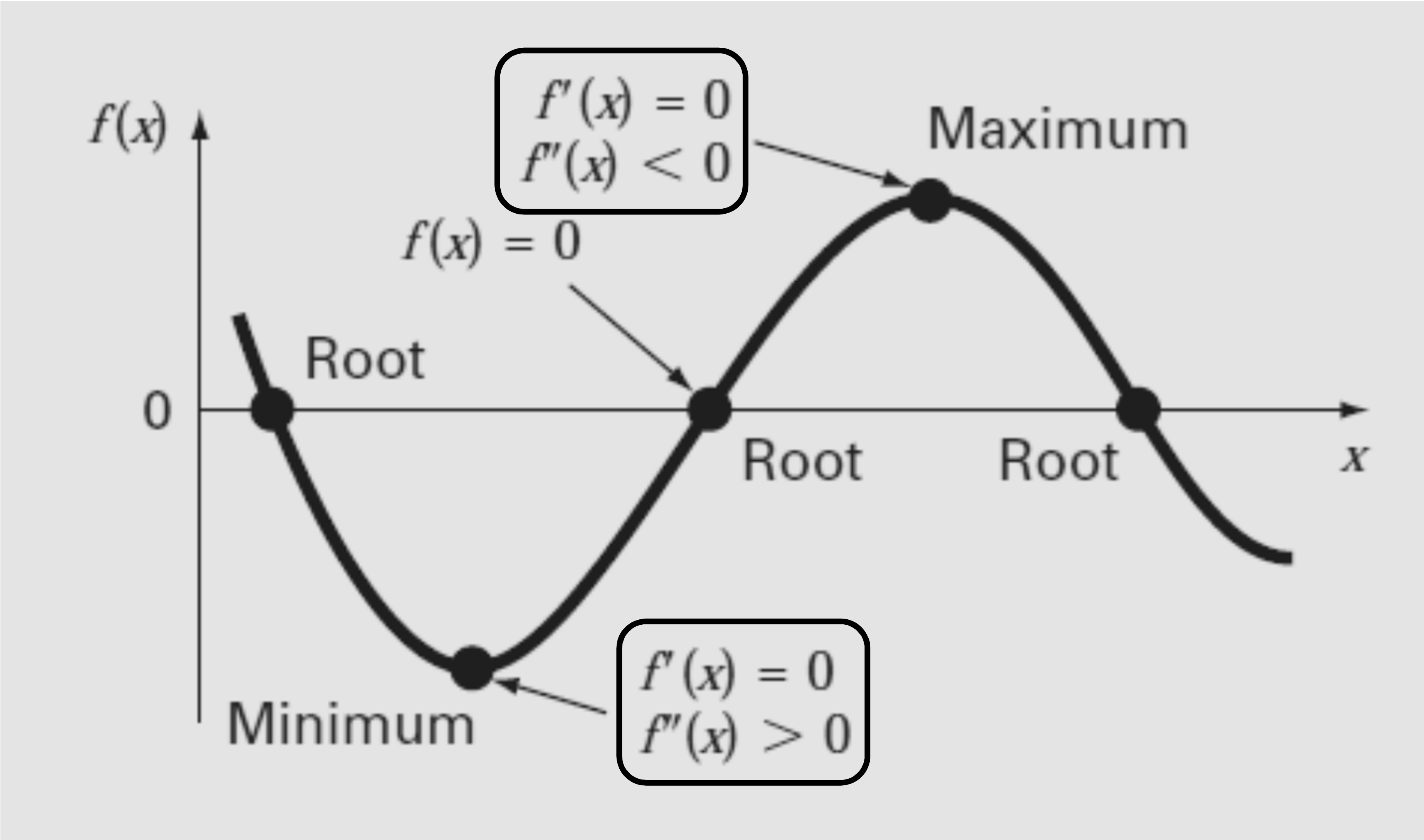
Optimization Problem

- Root location and Optimization are related in the sense that both involve guessing and searching for a point on a function.

- Root location involves searching for zeros of a function or functions. In contrast, optimization involves searching for either the minimum or the maximum.

Optimization Problem



Optimization Problem

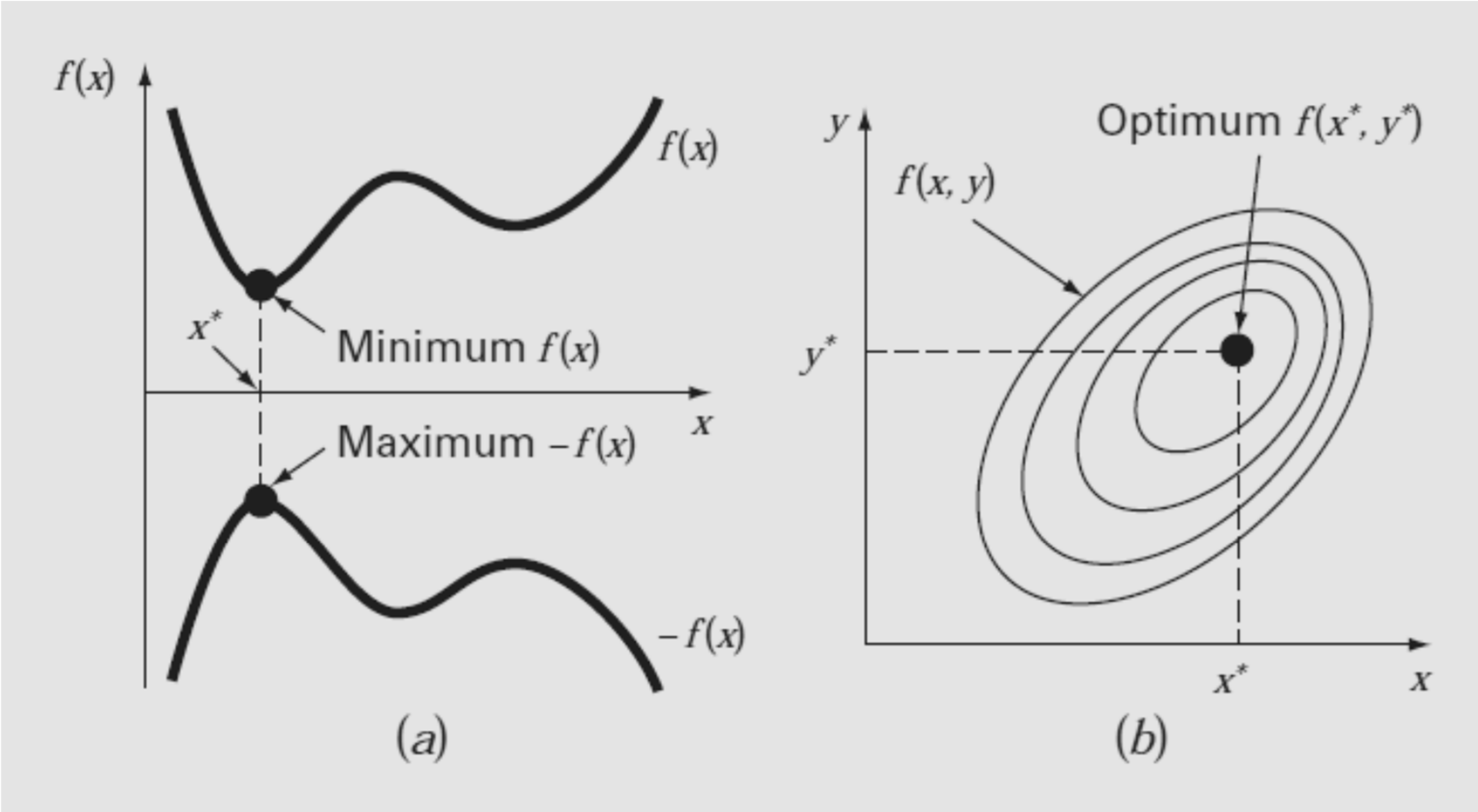
- An optimization or mathematical programming problem generally can be stated as:

Find x, which minimizes or maximizes f (x) subject to:

$$d_i(x) \le a_i \quad i = 1, 2, \ldots, m$$
$$e_i(x) = b_i \quad i = 1, 2, \ldots, p$$

Where x is an n-dimensional design vector, f (x) is the objective function, $d_i(x)$ are inequality constraints, $e_i(x)$ are equality constraints, and a_i and b_i are constants.

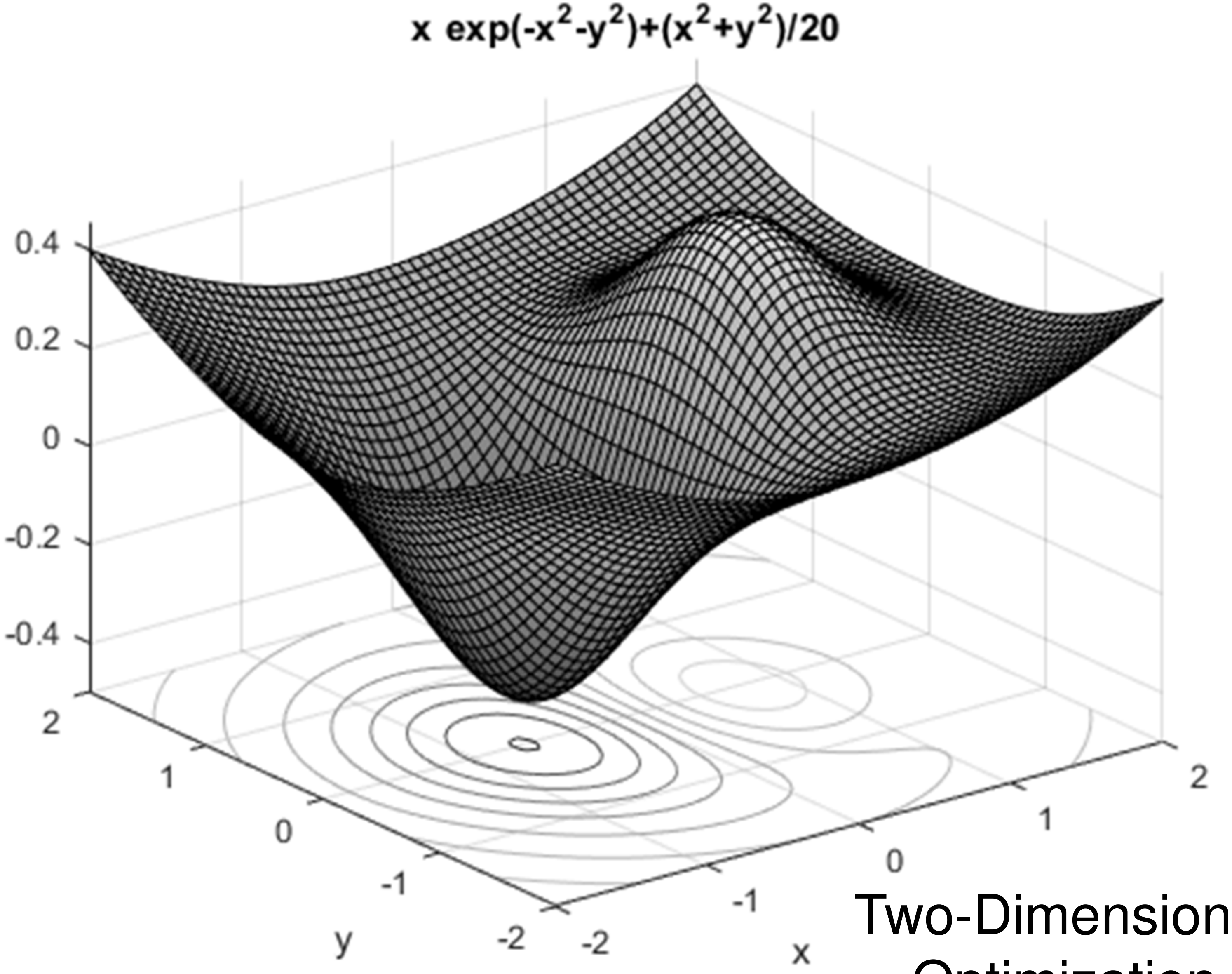
Optimization Problem



One-Dimensional Optimization

Two-Dimensional Optimization

Optimization Problem



Two-Dimensional Optimization

Optimization Problem

Types of Optimization Problems:

➨One-Dimensional Unconstrained Optimization.

(Golden-section search, parabolic interpolation, and Newton's method)

➨Multidimensional Unconstrained Optimization.

(Conjugate gradient, Newton's method, Marquardt's method, and quasi-Newton methods)

➨ Constrained Optimization

Optimization Problem

The General problem definition is:

Minimize the Objective Function $G(x_i)$

While satisfying the Constraints $H(x_i)=0.0$

In case There is no Constraints (H), or merging the constraints with the objective function, the problem is called unconstrained Optimization

Optimization Problem

- In case of maximizing the Objective Function $G(x_i)$, it is the same problem as minimizing - $G(x_i)$.

- To merge the constraints with the objective function to switch from constrained to unconstrained optimization, there are several methods such as penalty or Lagrange methods

$$G'(x_i)=G(x_i) + l\,[H(x_i)]^2$$

Where l is a large number

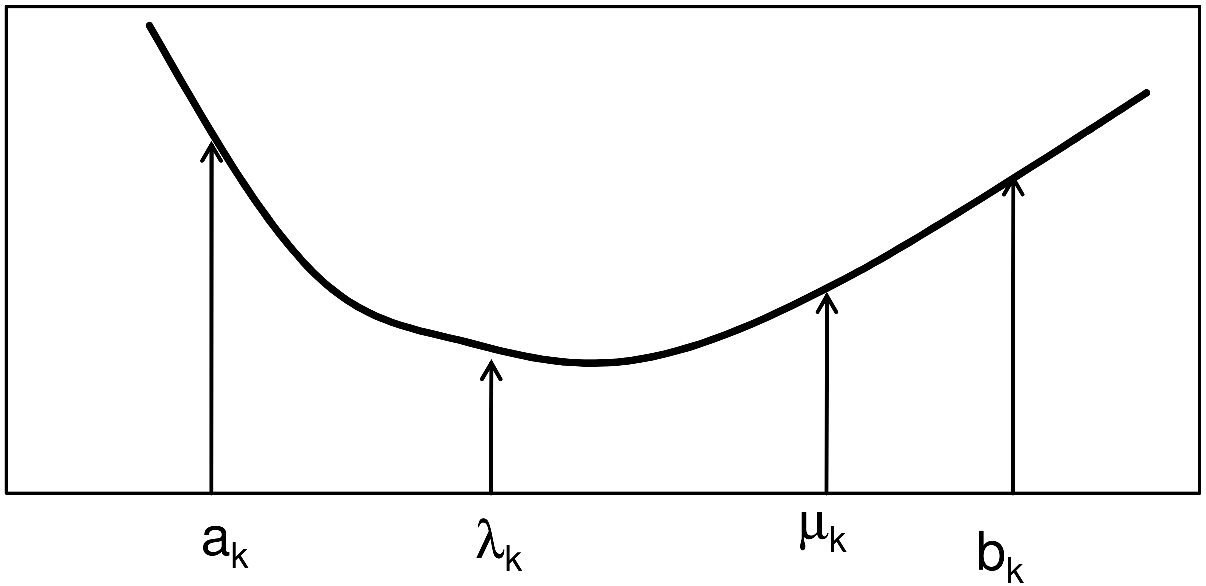
ONE-DIMENSIONAL OPTIMIZATION

Line Search

To minimize an objective function with one variable, line search can be categorized as:

- ➢ Line Search Without Using Derivatives:
  - ❖ Golden Section Method.
  - ❖ Fibonacci Search Method.
- ➢ Line Search Using Derivatives:
  - ❖ Bisection Method.
  - ❖ Newton’s Method.

Line Search : Golden Sections



If there is possible interval of uncertainty between a_k and b_k, we can try two inner values λ_k and μ_k then:

1- If $G(\lambda_k)$ is the smallest, then Minimum lies between a_k and μ_k.

2- If(μ_k) is the smallest, then Minimum lies between λ_k and b_k.

Line Search : Golden Sections

- The Golden Ratio:

$$\frac{\sqrt{5}-1}{2} = 0.61803\ldots$$

This ratio was employed for a number of purposes, including the development of the rectangle. These proportions were considered aesthetically pleasing by the Greeks. Among other things, many of their temples followed this shape.

Line Search : Golden Sections

➢ The Golden Ratio:

$$\frac{\sqrt{5}-1}{2} = 0.61803\ldots$$

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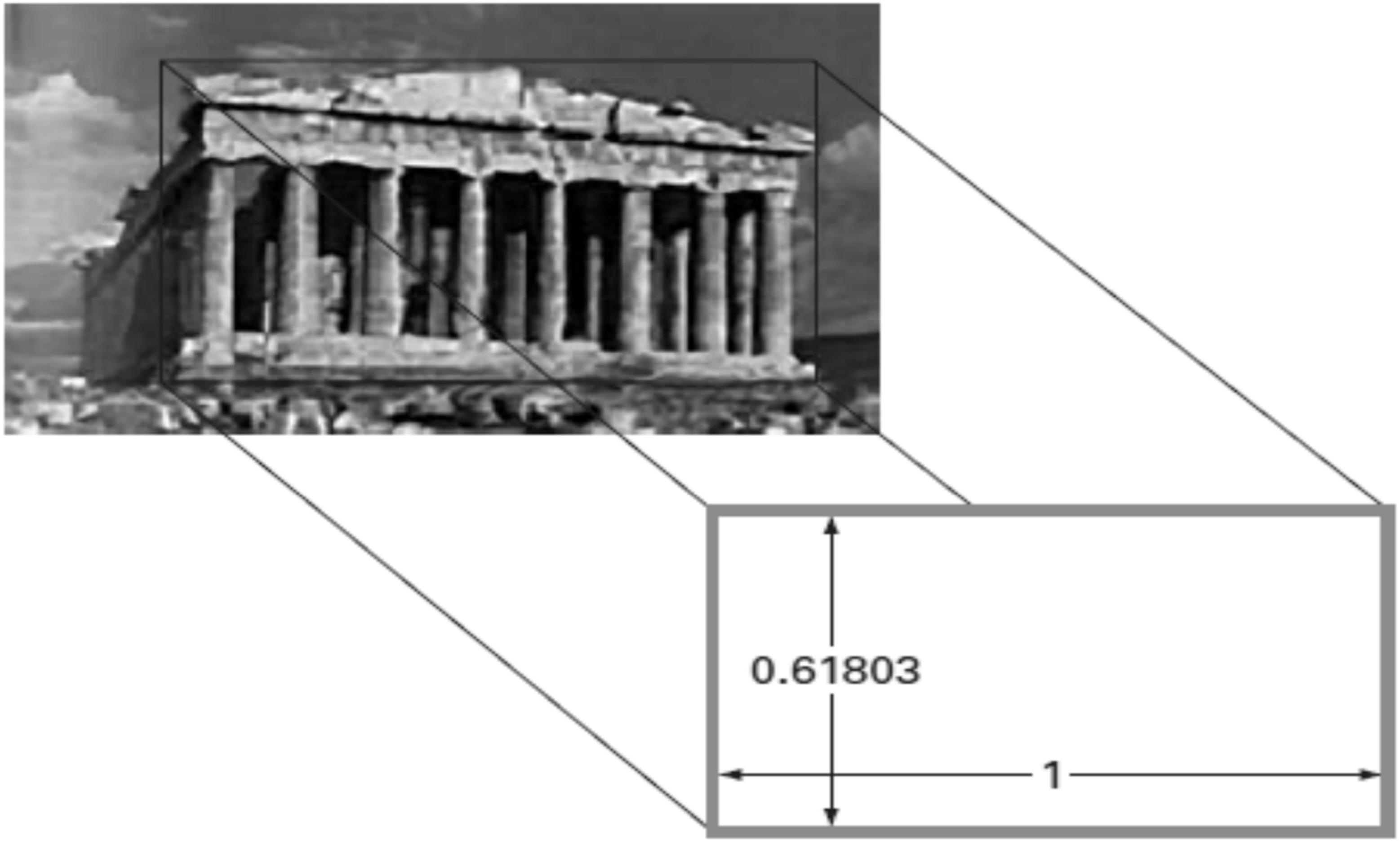


Line Search : Golden Sections

- To make use of the three values in the next iteration, we need to divide the interval between a_k and b_k using a certain ration (a) so that in the next iteration one of λ_k or μ_k will be reused.

- To Achieve this criteria, the following equations should be satisfied :

$$\lambda_k = a_k + (1-\alpha)(b_k - a_k)$$

$$\mu_k = a_k + \alpha(b_k - a_k)$$

$$\alpha^2 + \alpha - 1 = 0 \text{ then } \alpha = \sqrt{1.25} - 0.5 = 0.618034$$

Line Search : Golden Sections

EXAMPLE (1):

Find the minimum of:

$$G(x)=x^2+2x$$

Subject to

$$-3 \le x \le 5 \text{ (Possible uncertainty interval)}$$

Solution

By analytical means, the minimum of G is at x= -1. In order to obtain it with Golden sections see the following table:

Line Search : Golden Sections

K	a_k	b_k	$\lambda_{k=}a_k+(0.382)(b_k-a_k)$	$\mu_{k=}a_k+(0.618)(b_k-a_k)$	$G(\lambda_k)$	$G(\mu_k)$
1	-3.000	5.000	0.056	1.944	0.115	7.669
2	-3.000	1.944	-1.111	0.056	-0.988	0.115
3	-3.000	0.056	-1.833	-1.111	-0.306	-0.988
4	-1.833	0.056	-1.111	-0.666	-0.988	-0.888
5	-1.833	-0.666	-1.387	-1.111	-0.850	-0.988
6	-1.387	-0.666	-1.111	-0.941	-0.988	-0.997
7	-1.111	-0.666	-0.941	-0.836	-0.997	-0.973
8	-1.111	-0.836	-1.006	-0.941	-1.000	-0.997
9	-1.111	-0.941	-1.046	-1.006	-0.998	-1.000

Line Search (Fibonacci Search Method)

Fibonacci Mathematical Series:

1 ,1 ,2 ,3 ,5 ,8 ,13 ,21 ,34 ,55 ,89 ,144 ,233 , …$F_{i,}$

………F_N

- Each number after the first two represents the sum of the preceding two.
- An interesting property of the Fibonacci sequence relates to the ratio of consecutive numbers in the sequence; that is, 0/1 = 0, 1/1 = 1, 1/2 = 0.5, 2/3 = 0.667, 3/5 = 0.6, 5/8 = 0.625, 8/13 = 0.615
- As one proceeds, the ratio of consecutive numbers approaches the golden ratio!

Line Search (Fibonacci Search Method)

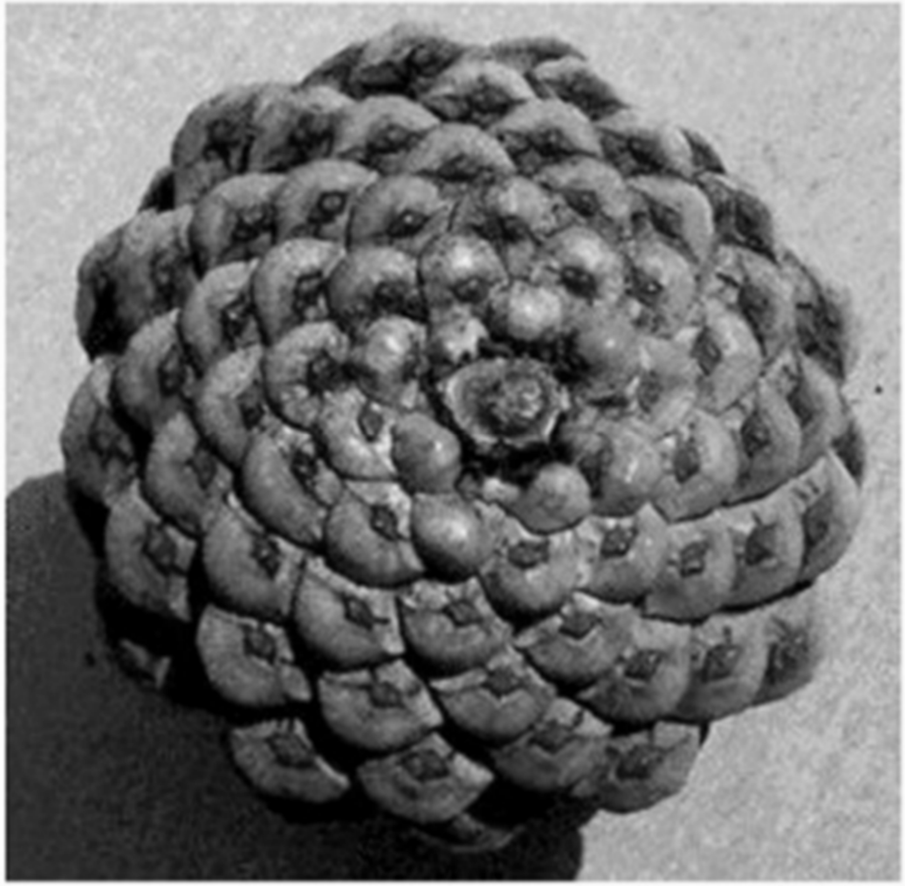
Figure: Columbine (left, 5 petals); Black-eyed Susan (right, 13 petals)

Figure: Shasta Daisy (left, 21 petals); Field Daisies (right, 34 petals)

Fibonacci Mathematical Series:

1 ,1 ,2 ,3 ,5 ,8 ,13 ,21 ,34 ,55 ,89 ,144 ,233 , …$F_{i,}$ ………F_N

Line Search (Fibonacci Search Method)



Bracts arranged in Fibonacci numbers of spirals

Fibonacci Mathematical Series:

1 ,1 ,2 ,3 ,5 ,8 ,13 ,21 ,34 ,55 ,89 ,144 ,233 , …$F_{i,}$ ………F_N

Line Search (Fibonacci Search Method)

Fibonacci used the following series of numbers to split the possible uncertainty interval (L):

1 ,1 ,2 ,3 ,5 ,8 ,13 ,21 ,34 ,55 ,89 ,144 ,233 , …$F_{i,}$ ………F_N

1- Define the allowable final uncertainty length D.

2- Number of Fibonacci series (N) could be defined from

$$F_N = L/D$$

Line Search (Fibonacci Search Method)

3- $\lambda_{k=} a_k + (F_{N-k-1} / F_{N-k+1})(b_k - a_k)$

$\mu_{k=} a_k + (F_{N-k} / F_{N-k+1})(b_k - a_k)$

4- Same organization shall be used as Golden Section Method

Line Search (Fibonacci Search Method)

EXAMPLE

Find the minimum of

$G(x)=x^2+2x$

Subject to

$-3 \le x \le 5$ (Possible uncertainty interval)

The acceptable final uncertainty interval length is 0.2

Solution

$F_N=(5-(-3))/.2=40$ then $N=9$

Line Search (Fibonacci Search Method)

F0	F1	F2	F3	F4	F5	F6	F7	F8	F9
1	1	2	3	5	8	13	21	34	55

K	a_k	b_k	$\lambda_k = a_k + (F_{8-k}/F_{10-k})(b_k - a_k)$	$\mu_k = a_k + (F_{9-k}/F_{10-k})(b_k - a_k)$	$G(\lambda_k)$	$G(\mu_k)$
1	-3.000	5.000	0.055	1.945	0.112	7.676
2	-3.000	1.945	-1.112	0.057	-0.988	0.118
3	-3.000	0.057	-1.833	-1.110	-0.307	-0.988
4	-1.833	0.057	-1.111	-0.664	-0.988	-0.887
5	-1.833	-0.664	-1.387	-1.110	-0.851	-0.988
6	-1.387	-0.664	-1.111	-0.940	-0.988	-0.996
7	-1.111	-0.664	-0.940	-0.835	-0.996	-0.973
8	-1.111	-0.835	-1.005	-0.940	-1.000	-0.996
9	-1.111	-0.940	-1.046	-1.005	-0.998	-1.000

Line Search (Bisection Method)

- In this method, the search is made for the zero value for the first derivative of the objective function.

- Bisection method could be used to find this zero value.

Line Search (Newoton's Method)

In this method, a quadratic approximation to the function "G" in the neighborhood of λ_k is described as Q as follows:

$$Q(\lambda) = G(\lambda_k) + G^{/}(\lambda_k)(\lambda - \lambda_k) + \frac{1}{2} G^{//}(\lambda_k)(\lambda - \lambda_k)^2$$

For minimum Q, the derivative of Q is equal zero. This yield to:

$$Q^{/}(\lambda) = G^{/}(\lambda_k) + \lambda G^{//}(\lambda_k)(\lambda - \lambda_k) = 0$$

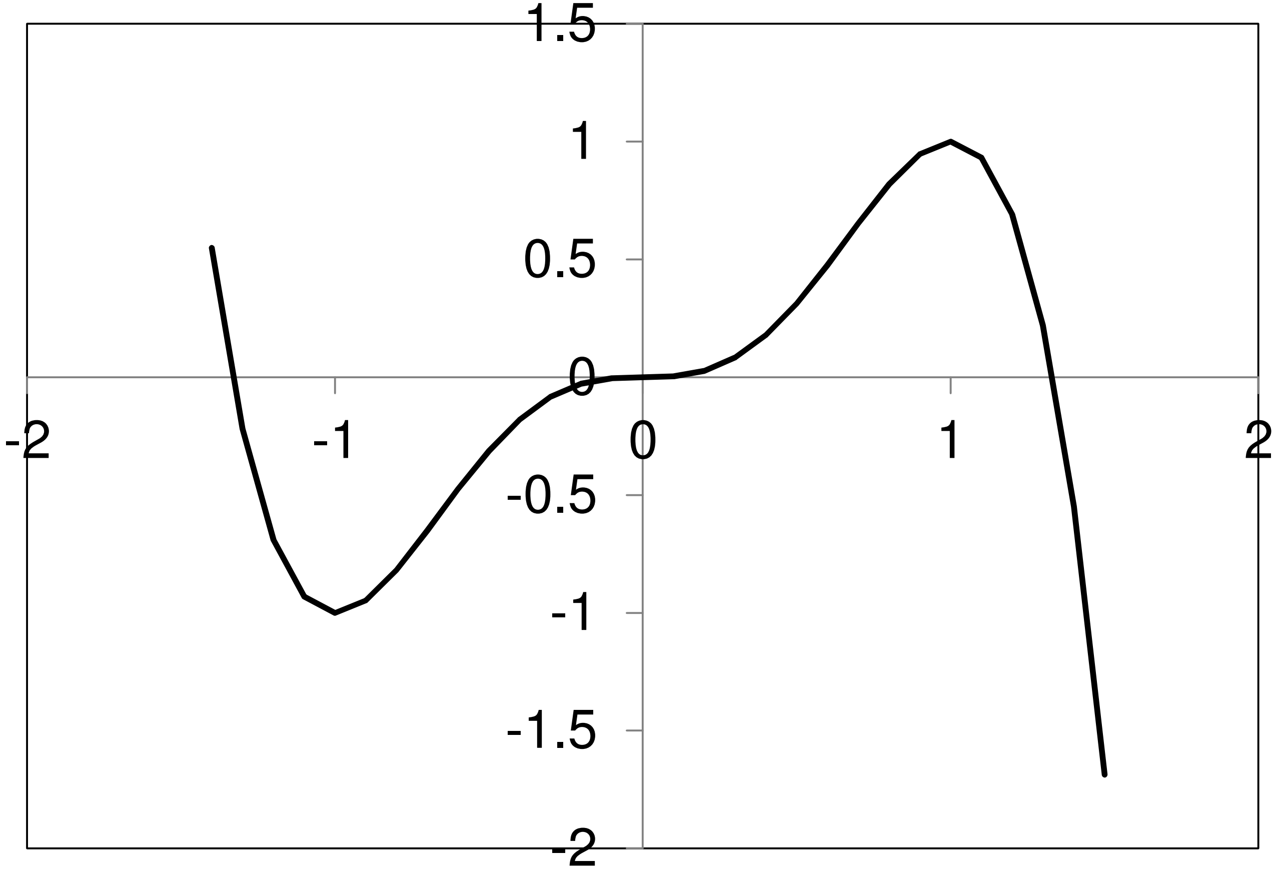
Then $Q^{/}$=0.0 at λ=λ_k

$$\lambda_{k+1} = \lambda_k - \frac{G^{/}(\lambda_k)}{G^{//}(\lambda_k)}$$

Line Search (Newoton's Method)

EXAMPLE

Find the minimum of $G(x) = \begin{cases} 4x^3 - 3x^4 & if \quad x \geq 0 \\ 4x^3 + 3x^4 & if \quad x < 0 \end{cases}$



Line Search (Newoton's Method)

EXAMPLE

The convergence depends on the initial value and how it is close to the actual minimum. If initial value of -0.7 is used, the convergence is achieved.

k	λ_k	$G^/$	$G^{//}$	λ_{k+1}
1	-0.7	1.764	0.84	-2.8
2	-2.8	-169	215	-2.01
3	-2.01	-49.2	97.51	-1.51
4	-1.51	-13.9	45.66	-1.2
5	-1.2	-3.56	23.31	-1.05
6	-1.05	-0.69	14.58	-1
7	-1	-0.06	12.22	-1
8	-1	-0	12	-1
9	-1	-0	12	-1
10	-1	0	12	-1

Line Search (Newoton’s Method)

EXAMPLE

If initial value of -0.6 is used, the convergence is not achieved.

k	I_k	$G^{/}$	$G^{//}$	I_{k+1}
1	-0.6	1.728	-1.44	0.6
2	0.6	1.728	1.44	-0.6
3	-0.6	1.728	-1.44	0.6
4	0.6	1.728	1.44	-0.6
5	-0.6	1.728	-1.44	0.6
6	0.6	1.728	1.44	-0.6
7	-0.6	1.728	-1.44	0.6
8	0.6	1.728	1.44	-0.6
9	-0.6	1.728	-1.44	0.6
10	0.6	1.728	1.44	-0.6

TWO-DIMENSIONAL OPTIMIZATION

Optimization Problem

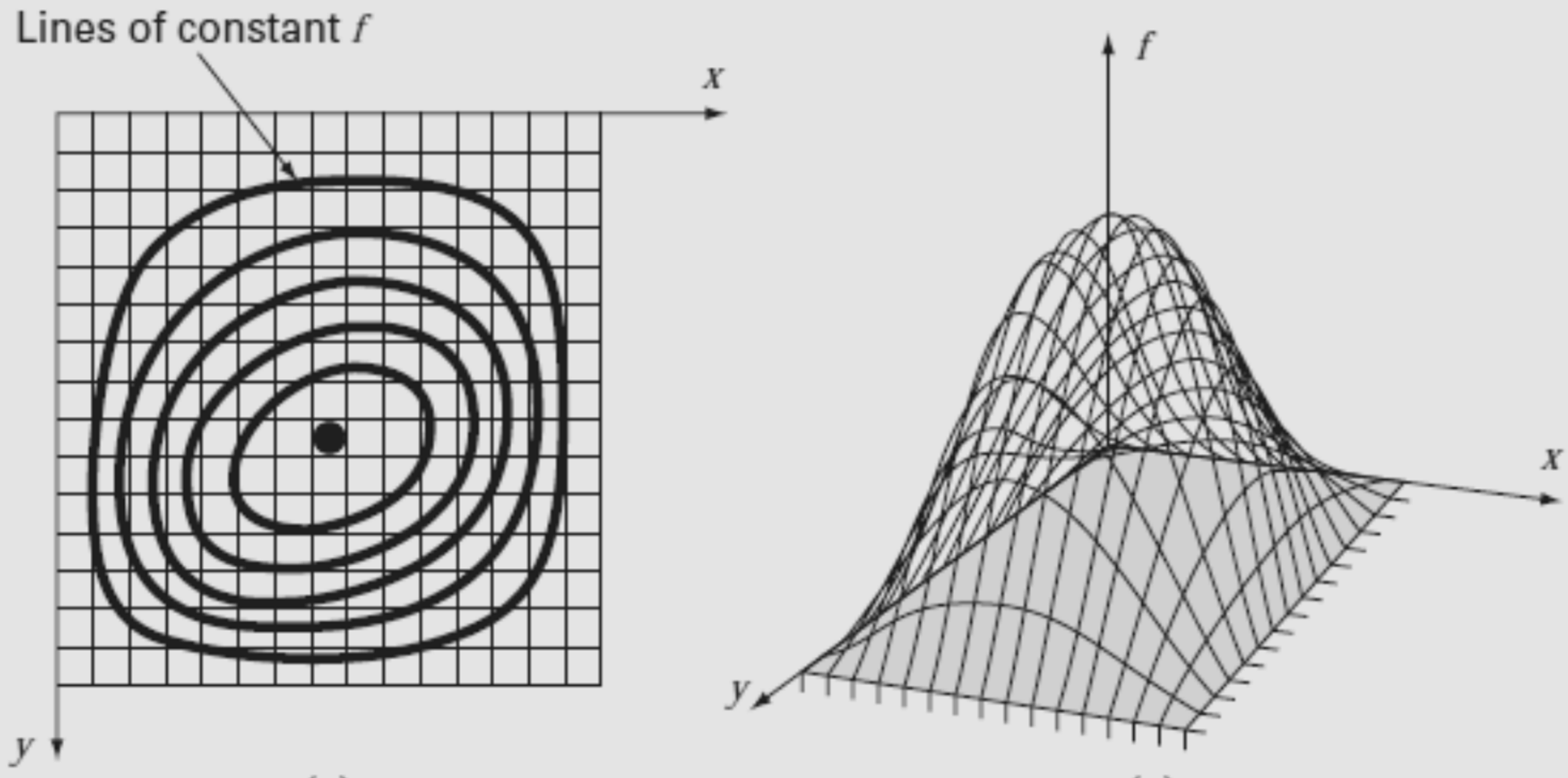
- The General problem definition is:

$$\text{Minimize the Objective Function } G(x_i)$$

$$\text{Where } i=1,n$$

- To minimize the objective function there are two approaches:
  - ✓ Multidimensional Search without using Derivatives.
  - ✓ Multidimensional Search using Derivatives.

Optimization Problem

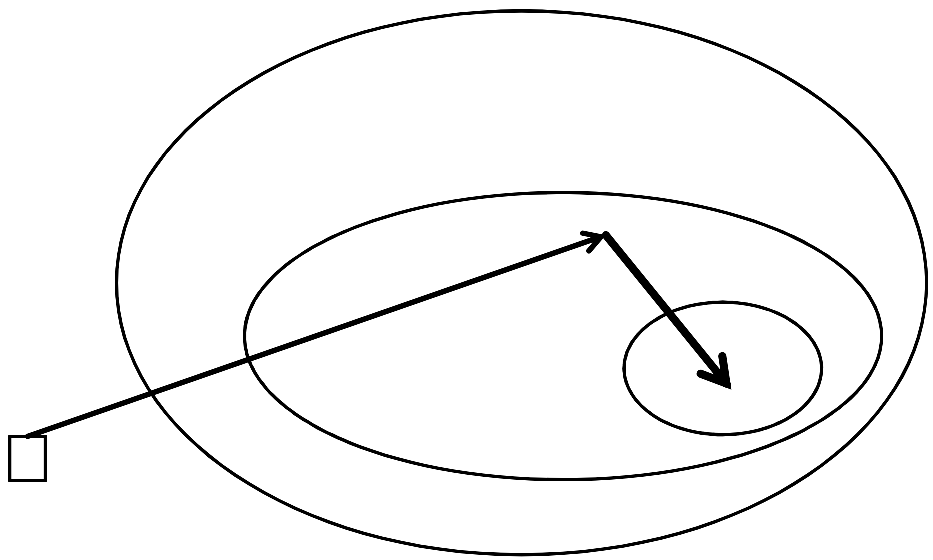


(a) (b)

Optimization Problem

To optimize a multi-dimensional Objective function, we need:

- A direction that we search for the minimum on it.
- Line Search to find the minimum value on this direction.



Multidimensional Search Without Using Derivatives

- Cyclic Method.

- Hooke and Jeeves.

- Rosenbrock

Cyclic Method

1- Chose a scalar value ε>0 to be used to terminate the algorithm (i.e. $\|X_{k+1} - X_k\| < \varepsilon$).

2- Choose an initial point $X_{1.}$

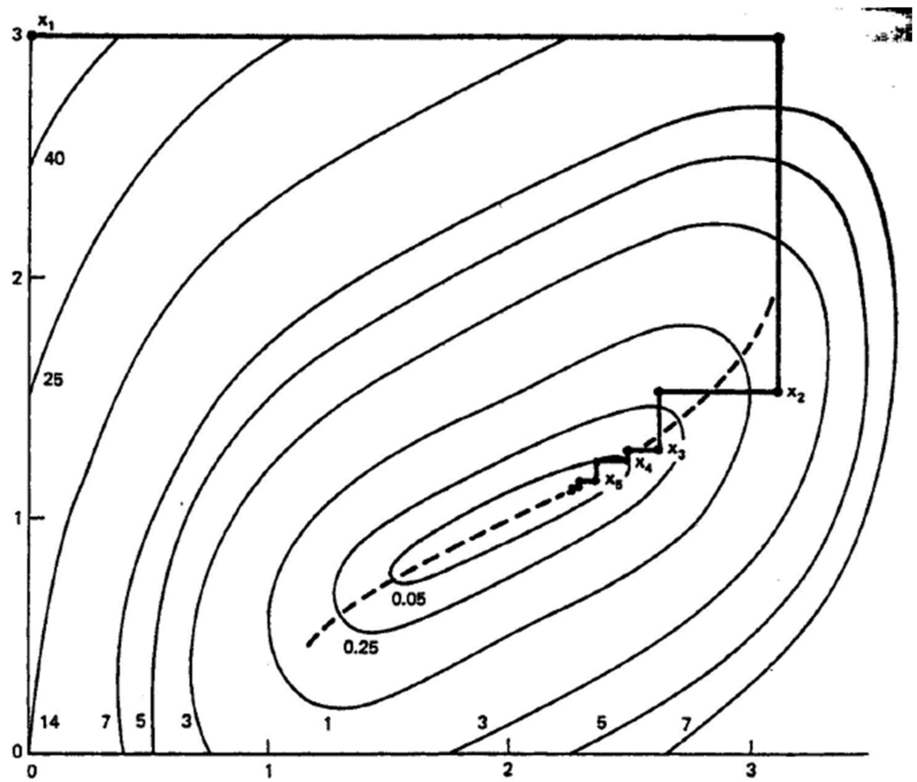
3- set $y_1 = X_1$ and K=1 (cycles Counter) and J=1 (Line search Counter)

4- Let λ be an optimal solution to minimize $G(y_j+\lambda\ d_j)$ where d_j is the coordinate directions (i.e. d_2 = (1 0 0 ...), d_2 =(0 1 0 0 ..), d_n =(0 0 .... 1))

5- Let $y_{j+1}= y_j+ \lambda\ d_j$

6- Repeat Step 4 using j+1, if j ≤ n otherwise goto step 7

7- Let $X_{k+1} = y_{n+1}$ and replace k by k+1 and repeat step 3 till $\|X_{k+1} - X_k\| < \varepsilon$



EXAMPLE

Minimize

$$G(X_i) = (x_1-2)^4 + (x_1-2x_2)^2 \qquad \text{Where } i=1,2$$

Table 8.6 Summary of Computations for the Cyclic Coordinate Method

Iteration k	$\mathbf{x}_k$ $f(\mathbf{x}_k)$	j	$\mathbf{d}_j$	$\mathbf{y}_j$	λ_j	$\mathbf{y}_{j+1}$
1	(0.00, 3.00)	1	(1.0, 0.0)	(0.00, 3.00)	3.13	(3.13, 3.00)
	52.00	2	(0.0, 1.0)	(3.13, 3.00)	−1.44	(3.13, 1.56)
2	(3.13, 1.56)	1	(1.0, 0.0)	(3.13, 1.56)	−0.50	(2.63, 1.56)
	1.63	2	(0.0, 1.0)	(2.63, 1.56)	−0.25	(2.63, 1.31)
3	(2.63, 1.31)	1	(1.0, 0.0)	(2.63, 1.31)	−0.19	(2.44, 1.31)
	0.16	2	(0.0, 1.0)	(2.44, 1.31)	−0.09	(2.44, 1.22)
4	(2.44, 1.22)	1	(1.0, 0.0)	(2.44, 1.22)	−0.09	(2.35, 1.22)
	0.04	2	(0.0, 1.0)	(2.35, 1.22)	−0.05	(2.35, 1.17)
5	(2.35, 1.17)	1	(1.0, 0.0)	(2.35, 1.17)	−0.06	(2.29, 1.17)
	0.015	2	(0.0, 1.0)	(2.29, 1.17)	−0.03	(2.29, 1.14)
6	(2.29, 1.14)	1	(1.0, 0.0)	(2.29, 1.14)	−0.04	(2.25, 1.14)
	0.007	2	(0.0, 1.0)	(2.25, 1.14)	−0.02	(2.25, 1.12)
7	(2.25, 1.12)	1	(1.0, 0.0)	(2.25, 1.12)	−0.03	(2.22, 1.12)
	0.004	2	(0.0, 1.0)	(2.22, 1.12)	−0.01	(2.22, 1.11)

Cyclic Method Convergence problems

1- If G is differentiable then the method converges to a stationary point

Cyclic Method Convergence problems

2- If G has ridges, then Cyclic method may not converge to the absolute minimum

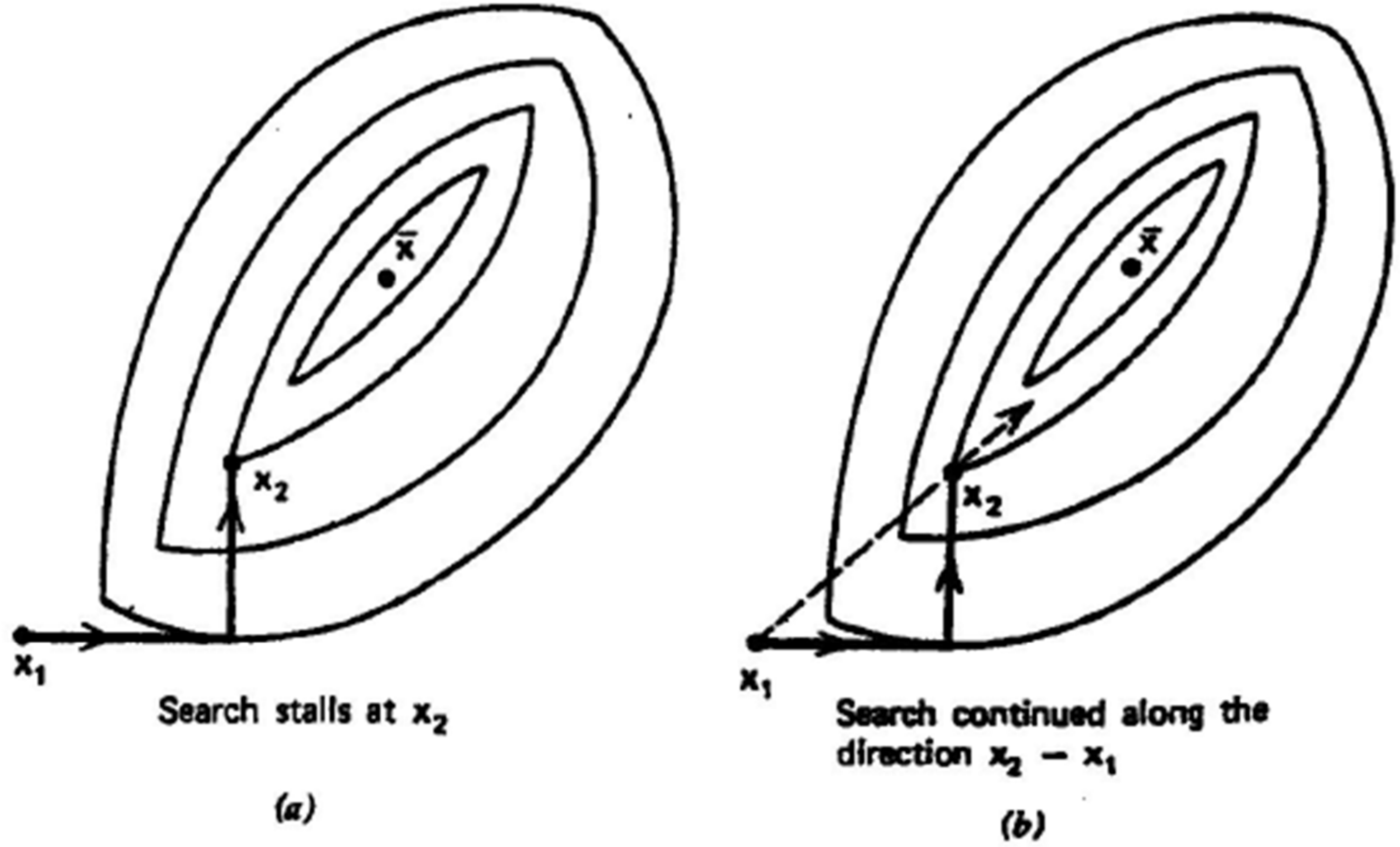


Figure 8.8 Illustration of the effect of a ridge.

Hooke and Jeeves

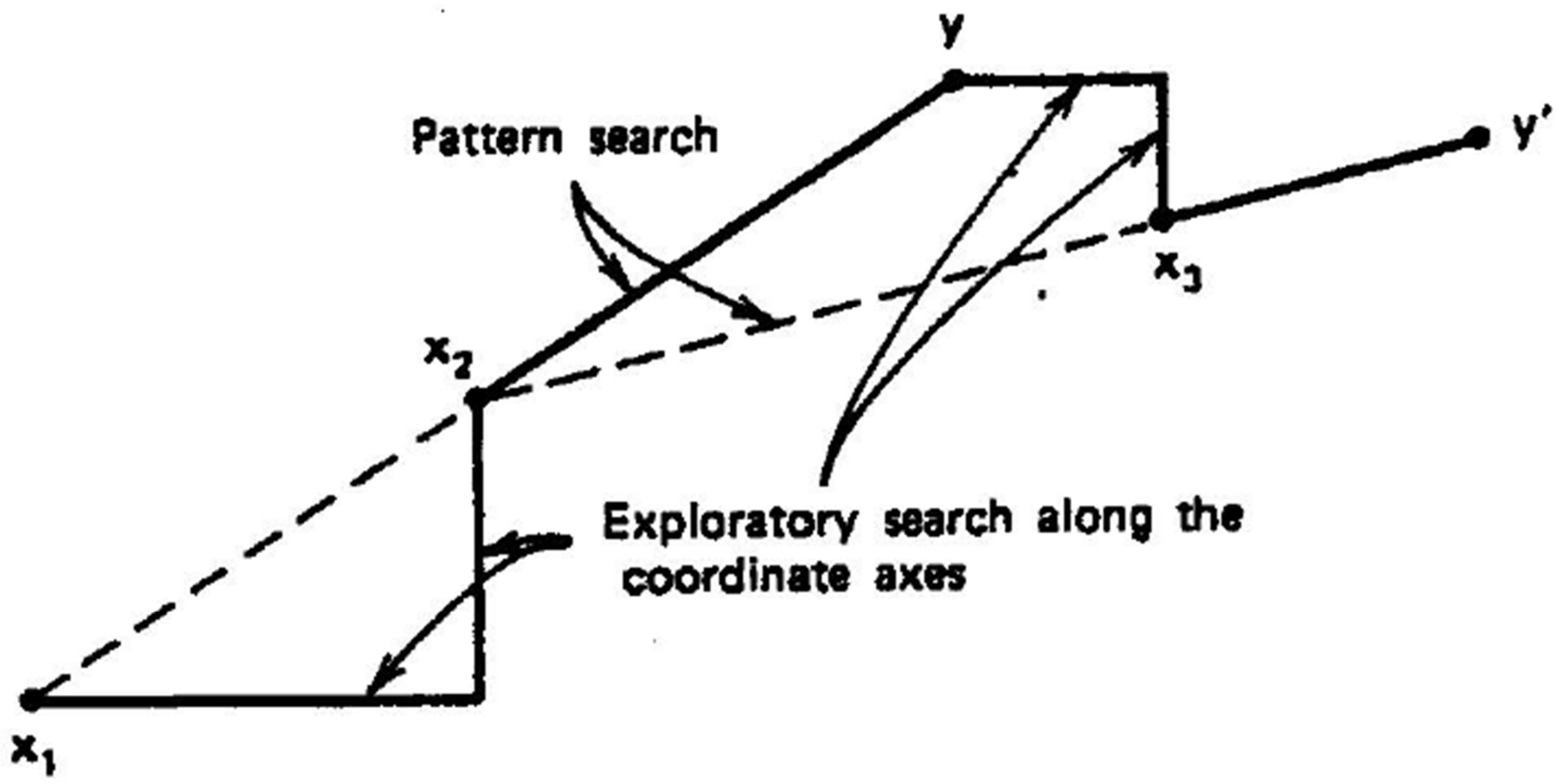
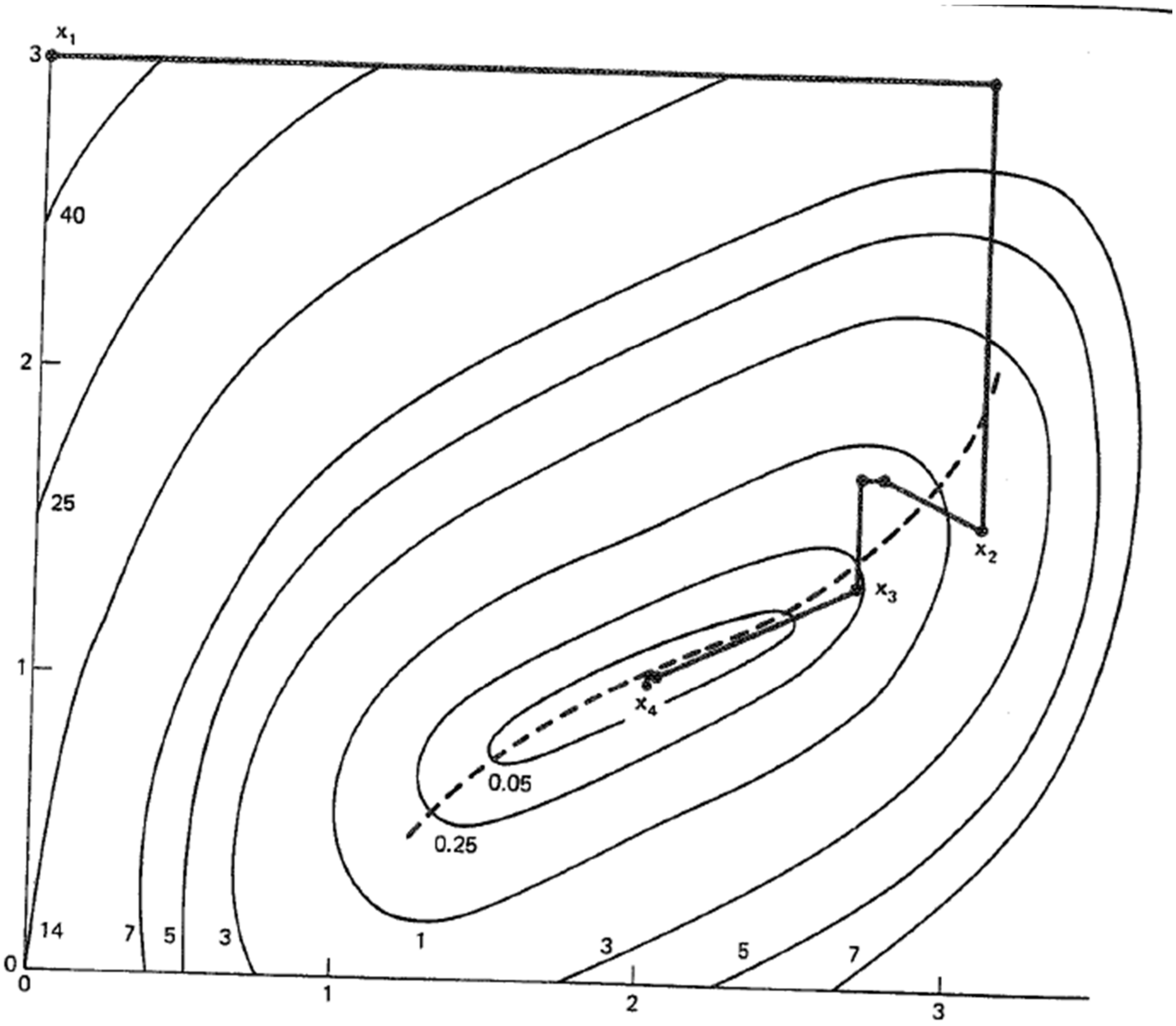


Figure 8.9 Illustration of the method of Hooke and Jeeves.

Hooke and Jeeves



Hooke and Jeevs

1- Choose a scalar value $\varepsilon>0$ to be used to terminate the algorithm (i.e. $\|X_{k+1} - X_k\| < \varepsilon$)

2- Choose an initial point X_1 and set k=1

3- Make one Cycle from the Cyclic Method to obtain X_{k+1}

4- Define the new search direction as $d = X_{k+1} - X_k$ and make a line search to get the minimum at y'

5- Set k=k+1 and repeat Step 3 using y' as an initial value

6- Keep repeating till $\|X_{k+1} - X_k\| < \varepsilon$

EXAMPLE

$G(X_i) = (x_1-2)^4 + (x_1-2x_2)^2$ Where i=1,2

Table 8.7 Summary of Computations for the Method of Hooke and Jeeves Using Line Searches

Iteration k	$\mathbf{x}_k$ $f(\mathbf{x}_k)$	j	$\mathbf{y}_j$	$\mathbf{d}_j$	λ_j	$\mathbf{y}_{j+1}$	$\mathbf{d}$	$\hat{\lambda}$	$\mathbf{y}_3 + \hat{\lambda}\mathbf{d}$
1	(0.00, 3.00)	1	(0.00, 3.00)	(1.0, 0.0)	3.13	(3.13, 3.00)	—	—	—
	52.00	2	(3.13, 3.00)	(0.0, 1.0)	−1.44	(3.13, 1.56)	(3.13, 1.44)	−0.10	(2.82, 1.70)
2	(3.13, 1.56)	1	(2.82, 1.70)	(1.0, 0.0)	−0.12	(2.70, 1.70)	—	—	—
	1.63	2	(2.70, 1.70)	(0.0, 1.0)	−0.35	(2.70, 1.35)	(−0.43, −0.21)	1.50	(2.06, 1.04)
3	(2.70, 1.35)	1	(2.06, 1.04)	(1.0, 0.0)	−0.02	(2.04, 1.04)	—	—	—
	0.24	2	(2.04, 1.04)	(0.0, 1.0)	−0.02	(2.04, 1.02)	(−0.66, −0.33)	0.06	(2.00, 1.00)
4	(2.04, 1.02)	1	(2.00, 1.00)	(1.0, 0.0)	0.00	(2.00, 1.00)	—	—	—
	0.000003	2	(2.00, 1.00)	(0.0, 1.0)	0.00	(2.00, 1.00)			
5	(2.00, 1.00)								
	0.00								

Multidimensional Search Using Derivatives

1- Steepest Descent Method

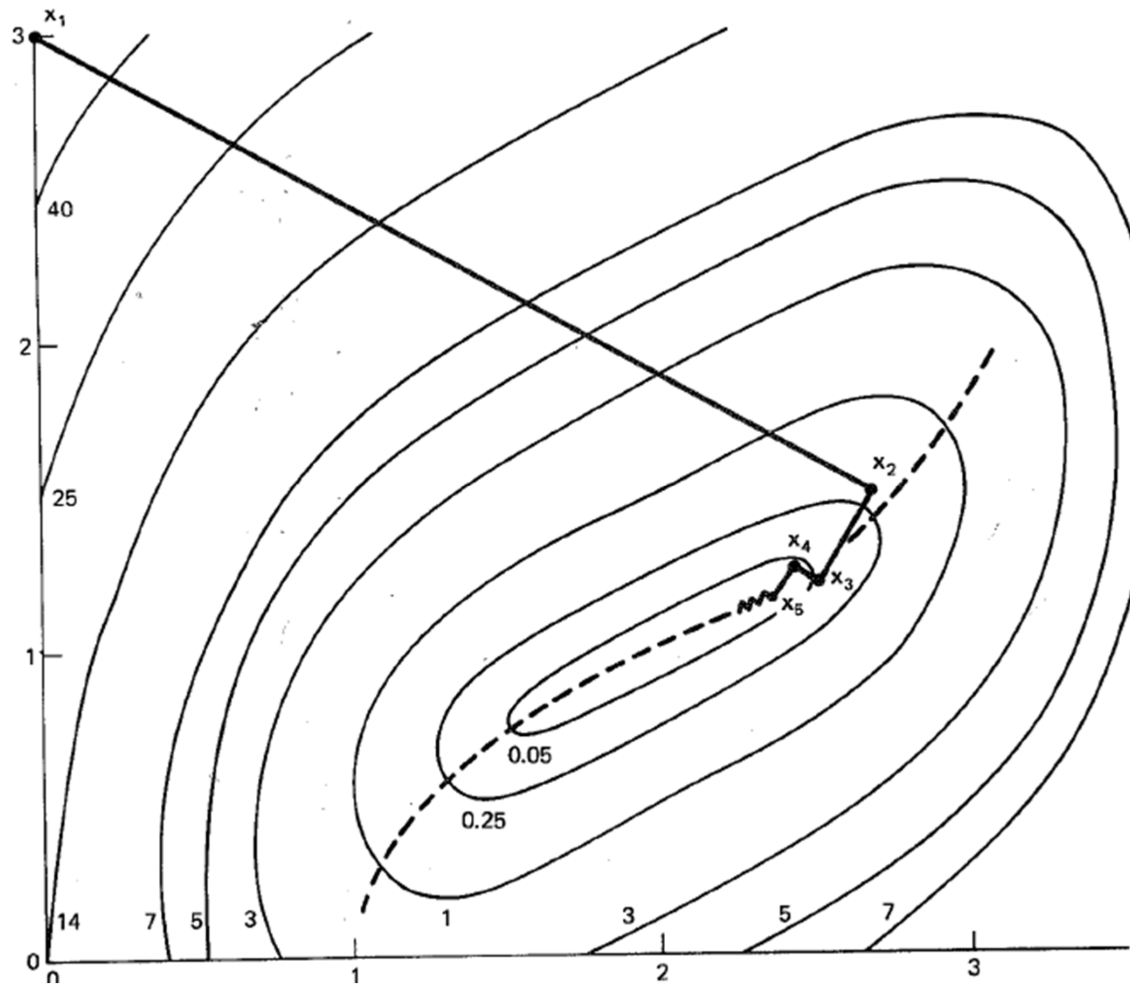
2- Newton Method

3- Conjugate Direction Method

Steepest Descent Method

1- Line search will take the following direction each global iteration

$$d = -\frac{\nabla G(x_k)}{\left\|\nabla G(x_k)\right\|}$$



Note:

$$\nabla G(x_k) = \begin{pmatrix} \frac{\partial G}{\partial x_1} \\ \frac{\partial G}{\partial x_2} \\ \cdot \\ \cdot \\ \frac{\partial G}{\partial x_n} \end{pmatrix}$$

EXAMPLE

$G(X_i) = (x_1-2)^4 + (x_1-2x_2)^2$ Where i=1,2

Table 8.11 Summary of Computations for the Method of Steepest Descent

Iteration k	$\mathbf{x}_k$ $f(\mathbf{x}_k)$	$\nabla f(\mathbf{x}_k)$	$\lVert \nabla f(\mathbf{x}_k) \rVert$	$\mathbf{d}_k = -\nabla f(\mathbf{x}_k)$	λ_k	$\mathbf{x}_{k+1}$
1	(0.00, 3.00) 52.00	(−44.00, 24.00)	50.12	(44.00, −24.00)	0.062	(2.70, 1.51)
2	(2.70, 1.51) 0.34	(0.73, 1.28)	1.47	(−0.73, −1.28)	0.24	(2.52, 1.20)
3	(2.52, 1.20) 0.09	(0.80, −0.48)	0.93	(−0.80, 0.48)	0.11	(2.43, 1.25)
4	(2.43, 1.25) 0.04	(0.18, 0.28)	0.33	(−0.18, −0.28)	0.31	(2.37, 1.16)
5	(2.37, 1.16) 0.02	(0.30, −0.20)	0.36	(−0.30, 0.20)	0.12	(2.33, 1.18)
6	(2.33, 1.18) 0.01	(0.08, 0.12)	0.14	(−0.08, −0.12)	0.36	(2.30, 1.14)
7	(2.30, 1.14) 0.009	(0.15, −0.08)	0.17	(−0.15, 0.08)	0.13	(2.28, 1.15)
8	(2.28, 1.15) 0.007	(0.05, 0.08)	0.09			

NEWTON Method

1- Line search will take the following direction each global iteration

$$x_{k+1} = x_k - H(x_k)^{-1}\nabla G(x_k)$$

Note:

$$H(x_k) = \begin{pmatrix} \frac{\partial^2 G}{\partial x_1^2} & \frac{\partial^2 G}{\partial x_1 \partial x_2} & & \frac{\partial^2 G}{\partial x_1 \partial x_n} \\ \frac{\partial^2 G}{\partial x_2 \partial x_1} & \frac{\partial^2 G}{\partial x_2^2} & & \frac{\partial^2 G}{\partial x_2 \partial x_n} \\ & & & \\ \frac{\partial^2 G}{\partial x_n \partial x_1} & \frac{\partial^2 G}{\partial x_n \partial x_2} & & \frac{\partial^2 G}{\partial x_n^2} \end{pmatrix}$$

$$\nabla G(x_k) = \begin{pmatrix} \frac{\partial G}{\partial x_1} \\ \frac{\partial G}{\partial x_2} \\ . \\ . \\ \frac{\partial G}{\partial x_n} \end{pmatrix}$$

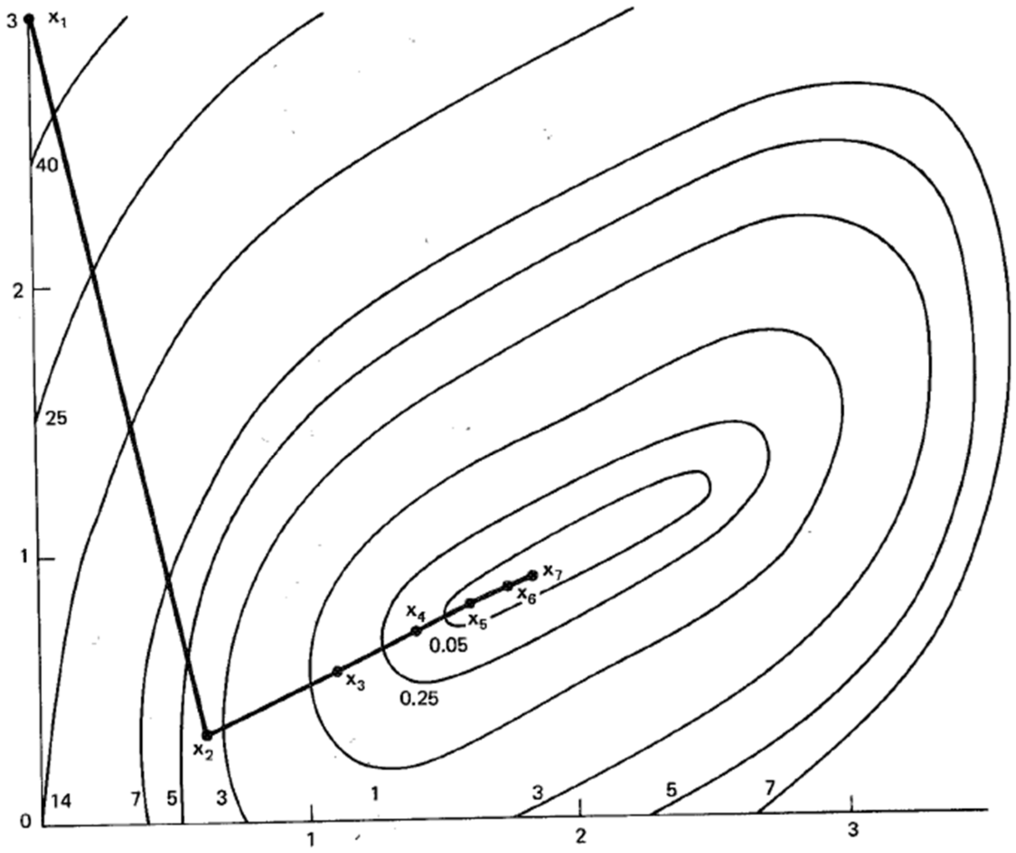


Figure 8.18 Illustration of the method of Newton.

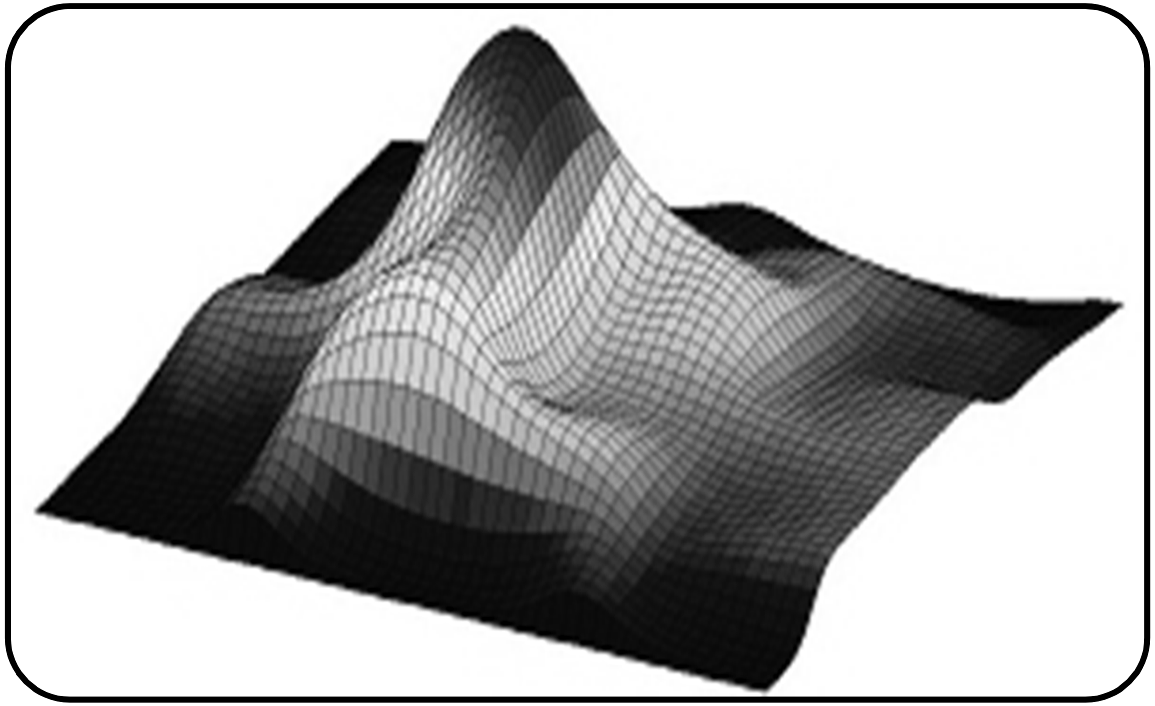
EXAMPLE

$$G(X_i) = (x_1-2)^4 + (x_1-2x_2)^2 \qquad \text{Where } i=1,2$$

Table 8.12 Summary of Computations for the Method of Newton

Iteration k	$\mathbf{x}_k$ $f(\mathbf{x}_k)$	$\nabla f(\mathbf{x}_k)$	$\mathbf{H}(\mathbf{x}_k)$	$\mathbf{H}(\mathbf{x}_k)^{-1}$	$-\mathbf{H}(\mathbf{x}_k)^{-1}\nabla f(\mathbf{x}_k)$	$\mathbf{x}_{k+1}$
1	(0.00, 3.00) 52.00	(−44.0, 24.0)	$\begin{bmatrix} 50.0 & -4.0 \\ -4.0 & 8.0 \end{bmatrix}$	$\frac{1}{384}\begin{bmatrix} 8.0 & 4.0 \\ 4.0 & 50.0 \end{bmatrix}$	(0.67, −2.67)	(0.67, 0.33)
2	(0.67, 0.33) 3.13	(−9.39, −0.04)	$\begin{bmatrix} 23.23 & -4.0 \\ -4.0 & 8.0 \end{bmatrix}$	$\frac{1}{169.84}\begin{bmatrix} 8.0 & 4.0 \\ 4.0 & 23.23 \end{bmatrix}$	(0.44, 0.23)	(1.11, 0.56)
3	(1.11, 0.56) 0.63	(−2.84, −0.04)	$\begin{bmatrix} 11.50 & -4.0 \\ -4.0 & 8.0 \end{bmatrix}$	$\frac{1}{76}\begin{bmatrix} 8.0 & 4.0 \\ 4.0 & 11.50 \end{bmatrix}$	(0.30, 0.14)	(1.41, 0.70)
4	(1.41, 0.70) 0.12	(−0.80, −0.04)	$\begin{bmatrix} 6.18 & 4.0 \\ -4.0 & 8.0 \end{bmatrix}$	$\frac{1}{33.44}\begin{bmatrix} 8.0 & 4.0 \\ 4.0 & 6.18 \end{bmatrix}$	(0.20, 0.10)	(1.61, 0.80)
5	(1.61, 0.80) 0.02	(−0.22, −0.04)	$\begin{bmatrix} 3.83 & -4.0 \\ -4.0 & 8.0 \end{bmatrix}$	$\frac{1}{14.64}\begin{bmatrix} 8.0 & 4.0 \\ 4.0 & 3.83 \end{bmatrix}$	(0.13, 0.07)	(1.74, 0.87)
6	(1.74, 0.87) 0.005	(−0.07, 0.00)	$\begin{bmatrix} 2.81 & -4.0 \\ -4.0 & 8.0 \end{bmatrix}$	$\frac{1}{6.48}\begin{bmatrix} 8.0 & 4.0 \\ 4.0 & 2.81 \end{bmatrix}$	(0.09, 0.04)	(1.83, 0.91)
7	(1.83, 0.91) 0.0009	(0.0003, −0.04)				

MatLAB Program
OPTIMIZATION TOOL



OPTIMIZATION TOOLBOX

- Useful for larger, more structured optimization problems.

- Sample functions include:

  Linprog, quadprog, fmincon, fminbnd

Use MATLab Help to know
the use of each function

Line Search (Fibonacci Search Method)

EXAMPLE

Find the minimum of

$G(x)=x^2+2x$

Subject to

$-3 \leq x \leq 5$ (Possible uncertainty interval)

The acceptable final uncertainty interval length is 0.2

OPTIMIZATION TOOLBOX

Unconstrained Optimization Example:

```
>>  x = fminbnd(@(x)(x.^2+2*x),-3,5)
```

Find minimum of single-variable function on fixed interval

Definition of Function

Range of Search

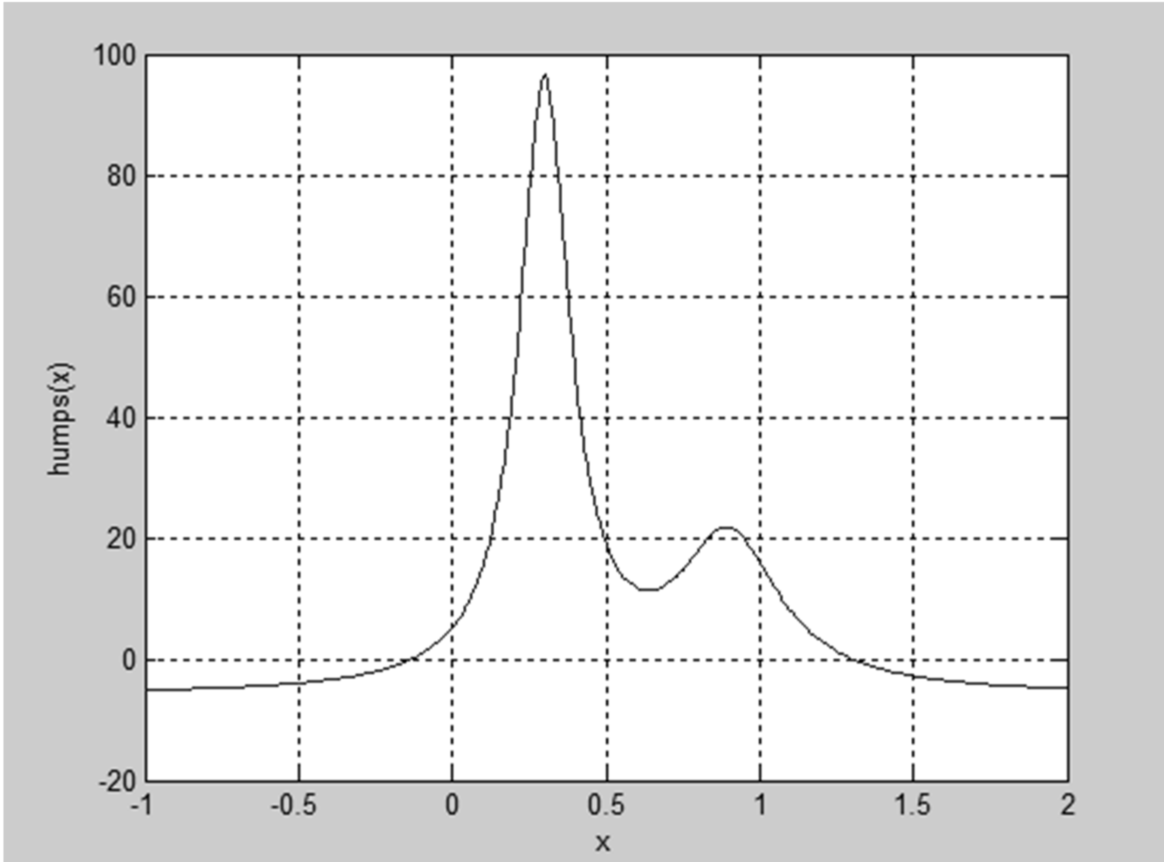
```
x =

    -1
```

Mathworks

OPTIMIZATION TOOLBOX

Unconstrained Optimization Example:

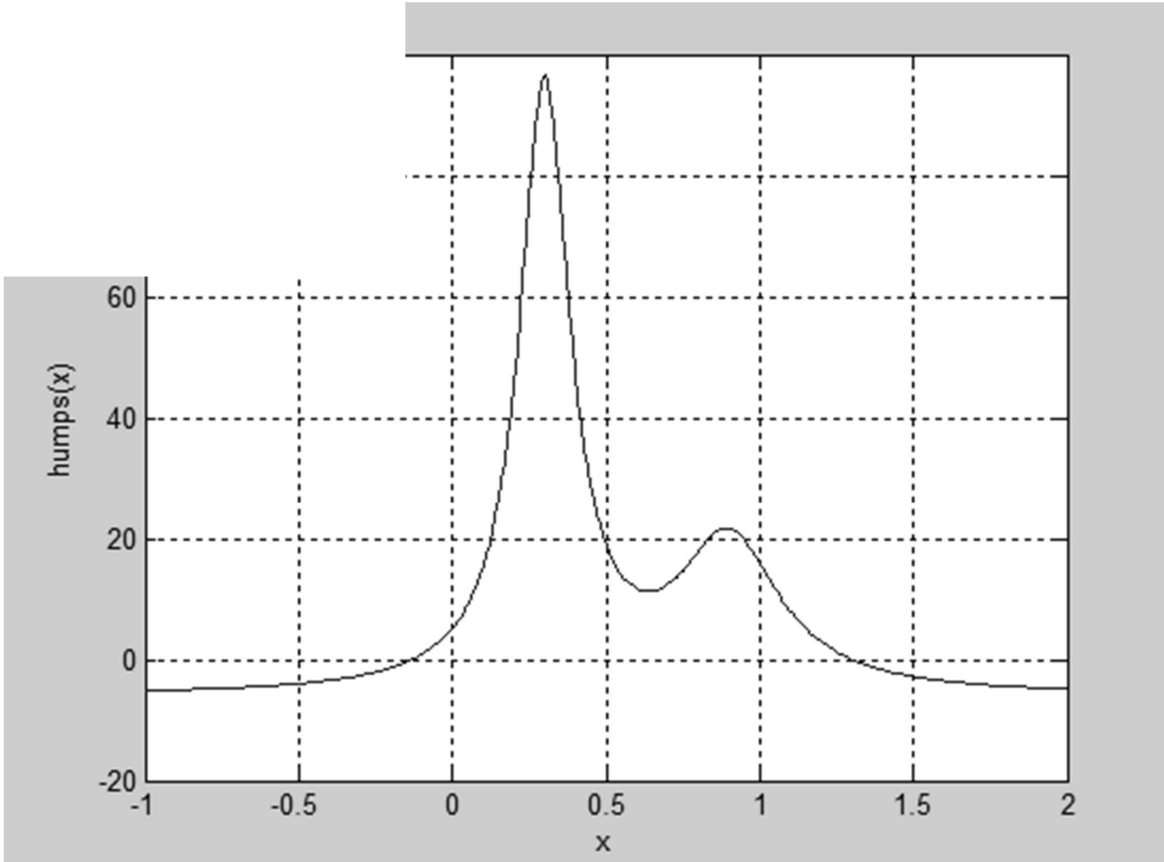
```
>> x = -1:.01:2;
y = humps(x);
plot(x,y)
xlabel('x')
ylabel('humps(x)')
grid on
```



OPTIMIZATION TOOLBOX

Unconstrained Optimization Example:

```
>> x = fminbnd(@humps,0.3,1)

x =

    0.6370
```



OPTIMIZATION TOOLBOX

Unconstrained Optimization Example:

Consider the problem of finding a minimum of the function:

$$x \exp(-(x^2 + y^2)) + (x^2 + y^2)/20.$$

OPTIMIZATION TOOLBOX

Unconstrained Optimization Example:

Plot the function to get an idea of where it is minimized.

```
f = @(x,y) x.*exp(-x.^2-y.^2)+(x.^2+y.^2)/20;
ezsurfc(f,[-2,2])
```

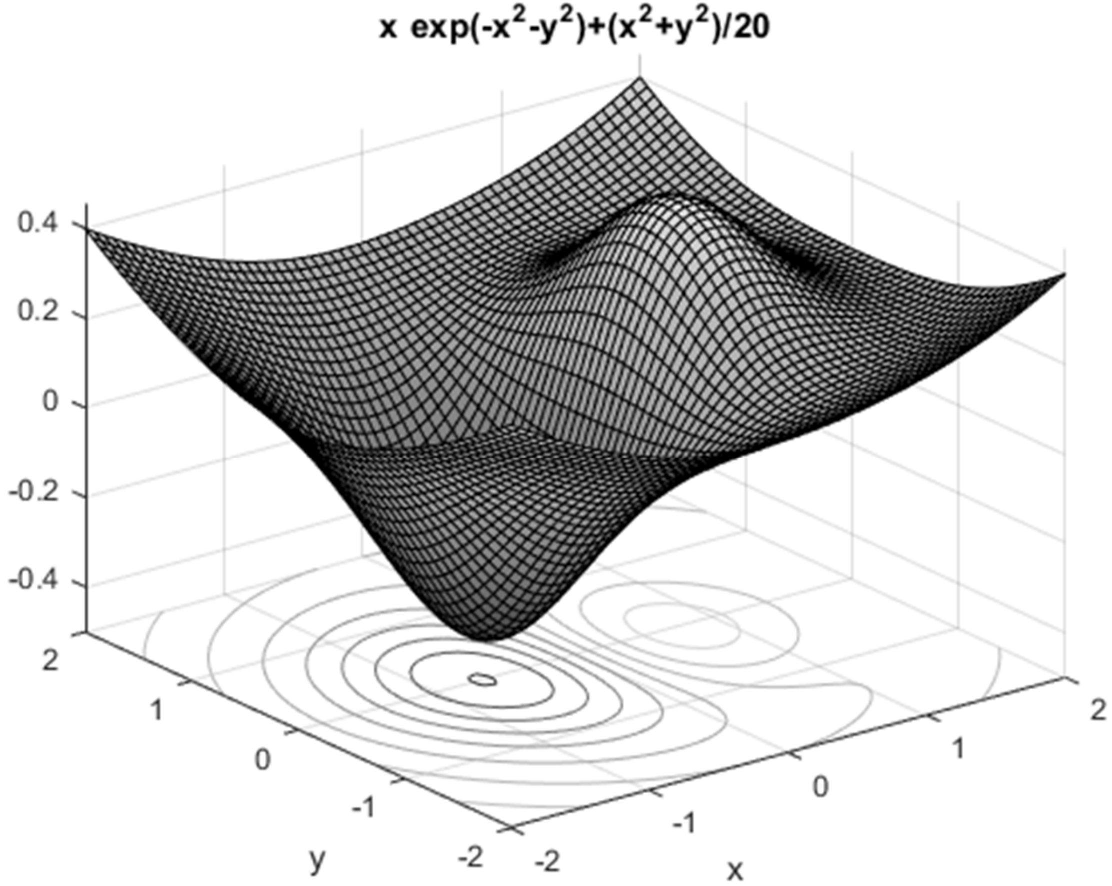
OPTIMIZATION TOOLBOX

Unconstrained Optimization Example:

Plot the function to get an idea of where it is minimized.

Minimum is at

(-0.5,0)



OPTIMIZATION TOOLBOX

Date	Topic
Tuesday 10 - 5	Optimization
Tuesday 17 - 5	Curve Fitting
Tuesday 24 - 5	Numerical Integration
Tuesday 31 - 5	Fourier Analysis